PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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Monday, 5th *March* 1883.

THE RIGHT HON. LORD MONCREIFF, President,
in the Chair.

The President read a communication from the Science and Art Department, South Kensington, London, in reference to the International Electrical Exhibition to be held in Vienna.

The following Communications were read:—

1. On the so-called Bicipital Ribs. By Professor William Turner.

In this paper an anatomical peculiarity was described which is occasionally found both in Man and the Cetacea. It is not due to a bifurcation of the shaft of a single rib at its vertebral end into two heads, but to the fusion of what ought to have been the shafts of two distinct ribs into a common body, and it invariably occurs at the apex of the thorax. Although the author had long been familiar with dried specimens from the human body in the Anatomical Museum of the University of Edinburgh, two cases which he now describes were the first that he had seen in the subject itself in the course of nearly thirty years' experience as a teacher of anatomy, and it is remarkable that they should both have occurred within a few months of each other. A third specimen occurred in a skeleton in the possession of one of his pupils, Mr Minas S. P. Aganoor.

A specimen from a large Cetacean was also shown and described. It formed a part of the skeleton of a *Balænoptera*, some of the bones of which were found in 1859, others in 1863, embedded in clay in

Christie's brickfield, in the carse land near the town of Stirling, and about 100 yards from the bed of the River Forth. They were lying in the "blue slink," from 13 to 14 feet below the present surface, and from 3 to 4 feet above the present high-water mark.

The following conclusions were stated:—

1. In both Man and the Cetacea cervical ribs are occasionally developed in connection with the 7th vertebra.
2. In both the cervical ribs may remain free or be fused with the 1st thoracic rib, so as to make it bicipital.
3. In Man a similar bicipital form may be due to fusion of the shafts of the 1st and 2nd thoracic ribs with each other at their vertebral ends, and it is probable that this may also occur in the Cetacea.
4. In either of the forms of fusion specified in 2 and 3, the two limbs, into which the vertebral end is divided, lie in different transverse planes, and the bifurcation is due to the partial fusion of two morphologically distinct rib-elements.
5. The presence of a cervical rib, or the bicipital form of the 1st rib, is only an individual peculiarity, and is not to be regarded as affording any evidence of either specific or generic difference.

The paper will appear in *extenso* in the *Journal of Anatomy and Physiology*, April, 1883.

2. Oscillations and Waves in an Adynamic Gyrostatic System. By Sir William Thomson.

3. On Gyrostatics. By the Same.

4. On the Dynamical Theory of Dispersion. By the Same.

Professor Tait laid upon the table a series of Photographs of Astronomical Instruments made at the new works of the Geneva Society for the Manufacture of Scientific Instruments. These photographs were sent by the Astronomer-Royal for Scotland.

BUSINESS.

The following candidates were balloted for, and declared duly elected Fellows of the Society:—Mr William Evans Hoyle, M.A., M.R.C.S.; Mr James Duncan Matthews; Mr James Greig Smith, M.A., M.B.; Mr John Archibald, M.B., C.M.; Mr Robert Rowand Anderson; Mr Andrew Gray.

Monday, 19th March 1883.

PROFESSOR DOUGLAS MACLAGAN, M.D., Vice-President, in the Chair.

The following Communications were read:—

1. On the Impossibility of Inverted Images in the Air.

By Edward Sang.

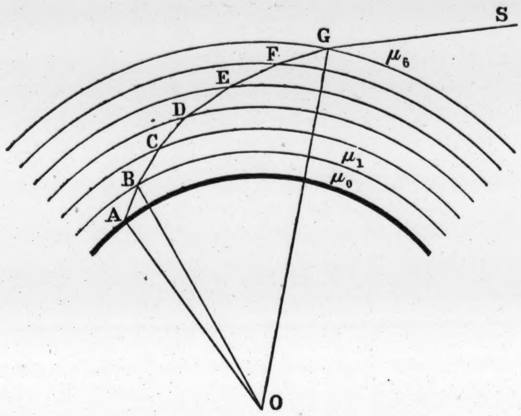
It is narrated that a physician, having given cogent reasons to show that there could be no recovery, and somewhat disconcerted by the return of sound health, yet maintained the cogency of his pathological arguments. The present may be a new edition of the same story; however, we shall advance the arguments, and leave it to futurity to tell whether there be any patient at all.

In order that an inverted image of any object be seen in the air, it is, in the first place, requisite that the light from that object, proceeding obliquely upwards, be bent again down to reach the eye of the observer; and, in the second place, that this retroflection occur in a determinate manner, so that the lights proceeding from different parts of the object may arrive in contiguous directions. These effects must be produced by the action of the air. The law according to which transparent media change the direction of light has been well known from direct experiment, and the index of refraction of air has been carefully measured.

In proceeding upwards, the light reaches air of gradually decreasing refractive power, so that the path is curved, the form of the curve depending on the law of diminution of the air's density, and also on the earth's roundness. The manner of the diminution of density is imperfectly known, and hence astronomers encounter great difficulty in investigating the amount of refraction for the stars; that amount, in fact, being computed from empirical formulæ. Yet there are relations among the heights, refractive powers and directions, very easily investigated; and, because of its great simplicity as well as for the sake of those who may not previously have looked at the matter or who may have been frightened by the display of algebraic symbols, I shall here give the analysis.

If the air were composed of concentric layers, each, within itself, of uniform density, the path of the light would be a series of straight lines, AB, BC . . . GS; refraction taking place at each junction.

If O be the earth's centre, and AB the path of the light in the first layer, the angle OAB, called by astronomers the nadir-distance,



is the incidence of the light upon the surface at A, while OBA measures the incidence of the same light upon the surface at B; and according to the most elementary laws of geometry, we have the proportion

$$\sin OAB : \sin OBA :: OB : OA.$$

If now μ_0, μ_1 be the indices of refraction for the first and second layers, the refraction at B must, according to the well-known law, be such that

$$\sin OBA : \sin OBC :: \mu_1 : \mu_0,$$

so that, on combining the two ratios, we have

$$\sin OAB : \sin OBC :: OB . \mu_1 : OA . \mu_0,$$

which proportion may also be represented by the equality

$$\sin OAB . OA . \mu_0 = \sin OBC . OB . \mu_1 .$$

The analogous equality holds for the path BC, and so along the whole course of the light, wherefore the continued product of the sine of the incidence, the radius vector and the index of refraction, is constant all along; that is to say, the equality

$$\sin OAB . OA . \mu_0 = \sin OGS . OG . \mu_6$$

holds good independently of the variations between the points A and G, and of the thickness or thinness of the supposed layers.

Hence if these three factors be known for any one point in the path, and if two of them be given for another point, the third factor corresponding to that point may be at once computed.

In this part of the theory of atmospheric refraction there is no difficulty; the astronomer's trouble is in discovering the geocentric angle AOG.

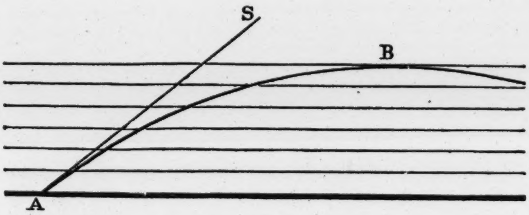
If the earth had been flat, the computations would have been still simpler; the product of the sine of the zenith distance by the index of refraction would have been constant, independently of the height, and the astronomer would have deduced the true from the apparent place of a star by a simple proportion.

M. Biot has given the index of refraction for air at 1.000 294. It varies with the pressure and temperature, and the formula

$$\mu = 1 + \frac{b}{100\,000}$$

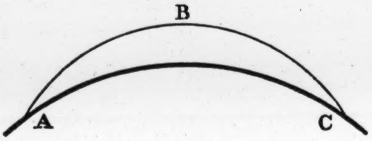
in which b stands for the height of the barometer in English inches, is sufficiently near for our purpose, is, perhaps, within the limits of error of the observations.

A ray of light rising obliquely from a point on the surface of a flat earth until it arrived at the upper limits of the air would have had the sine of its zenith distance augmented in the ratio of 1.000 000 to 1.000 294; and if the apparent zenith distance at the outset had been 88° 38′, that is, if its angle of elevation had been 1° 22′, whose secant is 1.000 294, the zenith distance at the upper surface of the air would have been 90°. No sun, moon, or star could have been seen at a lower altitude than 1° 22′. All light reaching the eye from a lower elevation must have come from some terrestrial object after having culminated as at B. The images of terrestrial objects would have been seen constantly in the air, but distorted by the retroflection,—the amount and character of the



distortion depending on the manner of diminution of the air's density.

In the actual case of the round earth, the phenomenon of a reflected image must necessarily lie between the above limit and horizontality.



If a ray of light leave the point C, culminate at B, and reach the eye at A with a zenith distance, which we shall denote by A, the incidence at B must be 90°, and therefore we must have the equality

$$OA \,.\, \sin A \,.\, 1.000\,294 = OB \,.\, \mu_B ,$$

or counting in English miles, and taking the earth's radius at 3960,

$$\sin A \times 3961.167 = OB \times \mu_B ;$$

now the smallest possible value of the index of refraction μ_B is unit, hence the maximum value of OB is

$$\sin A \times 3961.167 = OB ;$$

wherefore, since the greatest possible value of sin A is also unit, it follows that the utmost limit of OB, under any circumstances whatever, is 3961.167 miles; that is to say, no culmination of light in the atmosphere can take place at a height of more than 1·167 miles, or 6200 feet. But this light must reach the eye horizontally. If the angle A be less than 90°, the radius vector OB must be lessened in proportion to the sine; so that for $A = 88° \ 38'$, OB becomes 3960,—that is to say, no inverted image can be seen at an elevation of 1° 22′, unless the atmosphere have only an infinitely small thickness.

We are thus brought quite to home. Our researches do not extend into the regions of the aurora and magnetic arc; they are within the range to which our ordinary barometric measurements are known to apply with very considerable precision.

This result comes from the known index of refraction of air, and from the earth's dimensions, irrespective of all usual or unusual conditions of the air below the limit of culmination; but it requires that at that limit the air should altogether cease.

Let us examine the conditions needed in order that the light become horizontal at a lower level—say at the height of half a mile. In this case the index of refraction at B is obtained from the equality $\sin A \times 3961.167 = 3960.5 \times \mu_B$, or

$$\sin A \times 1.000\,168 = \mu_B,$$

which has possible solutions within two limits—one when $\mu_B = 1.000\,000$, in which case A would be 88° 57′, that is, the elevation would be 1°·03′; the other when A is 90°, giving $\mu_B = 1.000\,168$, which index of refraction would be due to a pressure of 17 inches of mercury. We have no idea of how such a rarefaction of the air at the height of only 2600 feet could be brought about.

To come still nearer home, let us propose an altitude of only one-tenth part of a mile, or 528 feet. We then find that with $A = 90°$, the index of refraction at the culminating point must be 1.000 269, which belongs to air under a pressure of 27 inches of mercury. This diminution of density cannot be due to the lessened pressure of the air, which, instead of 2·5 inches, would only give ·6, or about the quarter of the required reduction. In lowering the culminating point, we come nearer to the conditions of a flat earth, and therefore to the possibility or probability of a reflected image. Let us then inquire into the conditions when the light is close to the earth along its whole path, and tangent to the surface at C and at A.

If we put h for the height in miles, and $A = 90°$, the index of refraction is

$$\mu_B = \mu_O \times \frac{3960}{3960 + h},$$

or developing the fraction into series—

$$\mu_O - \mu_B = \mu_O \left\{ \frac{h}{3960} - \frac{h^2}{3960^2} + \&c. \right\}$$

Even at the absolute limit $h = 1·167$, the second and following terms of the series are too small to be of any account in our present inquiry, and thus we may hold that the diminution of the index would need to be at the rate of $\frac{\mu_0}{3260} = ·000\,2526$ for each mile of altitude. The diminution of the barometric pressure corresponding to this would be 25·26 inches of mercury for each mile, or ·00478 for each foot of rise. But that caused by the lessening of the atmospheric pressure is only ·00114, or less than the fourth part of what is needed; and therefore we conclude that no light can be retroflected in the usual condition of the atmosphere.

Inverted images, then, can only be seen when the air is in an unusual condition; there must be unusually light air above. Now, in these, as in all investigations on the subject, the air is assumed to be disposed in horizontal layers, each of uniform density; without such an arrangement no definite refraction can take place, no distinct image, whether distorted or not, can be formed.

The absolute need for smoothness of arrangement may easily be illustrated:—The sun's light is certainly reflected from the surface of the sea; yet we do not see an image of the sun in the water: we see only a confused brightness. When the air is quite still, the sea becomes smooth enough to give an image, which, however, the slightest breath of wind destroys. We cannot use a dish of water as an artificial horizon, we must cover even our trough of mercury with a glass screen to prevent the ripple caused by the wind. Here gravitation tends to produce and to preserve the evenness of the surface. Water is some seven hundred, mercury is ten thousand times heavier than air, and yet the friction of the air produces such disturbance.

Suppose then that we had a liquid as light as, or only a little heavier than air, and that we had poured this liquid into a flat dish. Paying no attention to the density of the wind which may blow upon it, let us think how calm the air would need to be that there may be no ripple on the surface. But let us add to this the consideration of the fact that the almost equality of the two densities deprives gravitation of its power to stratify, and we must admit that the slightest horizontal motion would be destructive of all smoothness.

Imagine a stratum of air of the requisite depth and in its usual state quite level on its upper surface, and let us place upon that a layer of light air of the requisite refractive power, we shall then have inverted images. But our strata are not liquids; they must be pressed upon to keep them from expanding, there must be air above them. If the superior air have its density conformable to the lower stratum so as to be in equilibrium with the general atmosphere beyond, it must be denser than the inserted layer, and the two would inevitably mix. If, on the other hand, the superior air be conformable to the upper layer its altitude must be greater than that of the general atmosphere which would press in to displace it. In neither case could there be repose.

But the question arises, "how is this layer of lighter air produced?" It cannot be from the sun's warmth in still weather, because then the heating is at the surface of the earth or of the water. The warmed air mixes with the cold above, ascending while the other descends, and giving rise to the too-well-to-the-astronomer-known boiling of the sun's edge.

The only other source of such warm air is in the south, whence the south-west wind brings it to us in gusts and squalls. At the oncoming of the breeze we may see changes in the appearance of distant objects on the horizon; they are displaced, distorted, horizontally, vertically, obliquely, according to the passing whims of Eolus. Seldom more than telescopic, they are as changeable as the squall itself. We can scarcely imagine that the wind should gently lay a warm coverlet over the quiet cold air of the north and leave it there in repose.

But, I shall be told, these images have been seen, your patient is alive and well, Professor Vince saw him from Ramsgate.

I have already shown that Vince's narrative is inconsistent with ordinarily known appearances, and I have pointed out that the same observer, after eight years' further experience as Professor of Astronomy and of Experimental Philosophy at Cambridge, was able to see through a telescope magnifying thirty times, a building fourteen miles away in all the beautiful proportions of near perspective. This feat could only have been performed by one totally ignorant of, or utterly careless of, the simplest laws of geometry and of optics.

Yet I needed not to have gone so far. When a picture is presented to us, we, without requiring an explanation, form an idea of what it means. It may be the picture of a horse, badly drawn, yet still we recognise the limner's meaning, he meant to have drawn a horse. So when I look at Vince's figure, I perceive that he meant to have drawn a sloop; he has succeeded in drawing a sloop such as neither I, nor you, nor any other person ever saw. But I see a great deal more, I recognise in the picture the unmistakable well-known features of an old acquaintance.

The picture is that of a sloop floating on a calm sea with its shadow in the water; the sharpness of the image is considerably overdone, very few persons have seen it so sharp. But the characteristic feature is there; the image and the ship are united at the water-line; it is a well known and understood appearance.

Oh! but Vince saw it up in the air!

When the sea is smooth, the reflection of the sky from the water and the light of the sky itself are so nearly balanced as to be undistinguished, hence the difficulty (sometimes amounting to impossibility) which the seaman has in bringing down the sun to the water-edge. He is unable to tell where the sea ends or the sky begins.

When the breath of a zephyr touches such a sea, it causes a dark space which is foreshortened into a narrow streak. Such streaks are apt to be mistaken for the true horizon, and the sailor-apprentice may find himself wrong in his latitude; but the mate or the captain has been on his guard.

Here, then, is the whole diagnosis of Vince's drawing:—He had seen a sloop floating on a very calm sea, the captain had taken the opportunity to air his canvas, some stray spars or some seaweed had been on the edge of a ripple in-shore, and the Professor's powerful imagination had manufactured wonders out of the vision.

2. On the Thermo-electric Positions of pure Rhodium and Iridium. By Professor Tait.

3. Observations on the Growth of Wood in Deciduous and Evergreen Trees. By the late Sir R. Christison, Bart., and Dr Christison.

4. The Variation of Temperature, with Sun-Spots. By Mr A. Buchan.

Monday, 2nd April 1883.

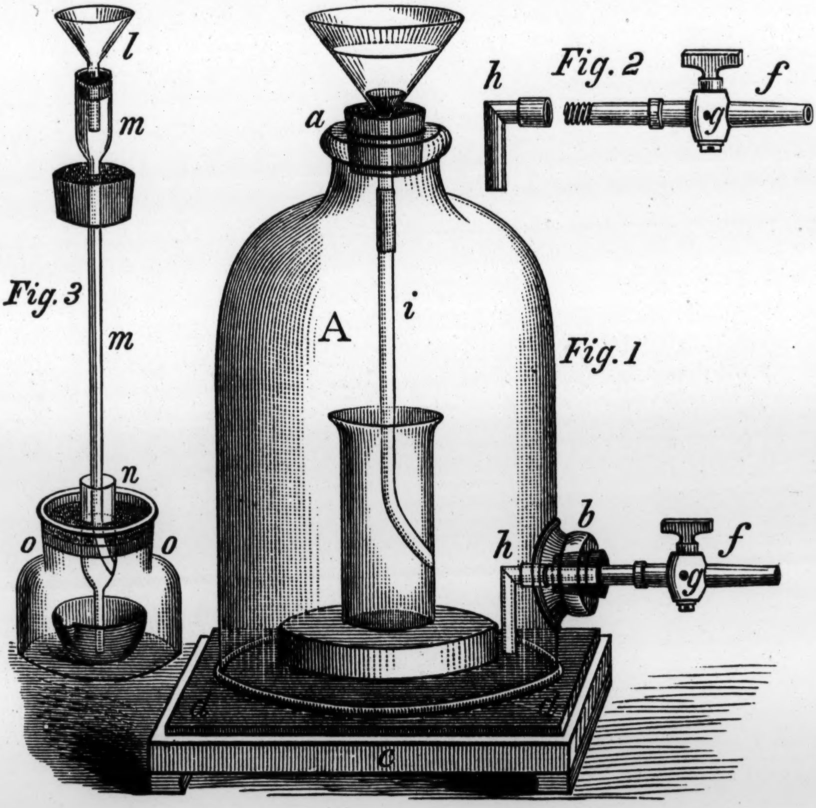
Mr JOHN MURRAY in the Chair.

The following Communications were read:—

1. On some Laboratory Arrangements. By Dr John Gibson.

Improved Method of Filtration by Diminished Pressure.

This apparatus will be best understood by a reference to the figures. The bell-jar A (fig. 1) rests, during filtration, on a square block of hard wood *c*, 25 cm. square by 25 mm. thick, over which is laid a sheet of vulcanised rubber *d* of good quality, and not less than 3 mm. thick. In the centre is a circular disk *e* also of hard wood, 13 cm. in diameter and about 20 mm. thick, which is held in position by four strong brass screws which pass down through the rubber into the square block below. In order to prevent air



leaking through the holes thus made, they are rendered quite air-tight by embedding the screw heads, which are sunk slightly below the surface of *e*, in red lead, and by laying, previous to screwing down, layers of red lead on both sides of the rubber sheet. The red lead should not, however, spread beyond the disk *e* on either side of the rubber sheet, so that the latter lies quite free except where it is held down by the disk. In *b* is fitted a single-bore rubber cork holding the brass tap *f*. This tap is simply an ordinary brass tap converted into a three-way tap by boring a hole *g* through one outer wall and through one wall of the plug, which enables the operator to establish communication between

(1) The bell-jar and pump.
(2) The outer air and both bell-jar and pump.
(3) The outer air and pump, the bell-jar being shut off.
(4) The outer air and bell-jar, the pump being shut off.

After fitting in the cork and tap, the knee-piece *h* (fig. 2) is screwed on. The object of this knee-piece will be explained later on. If the mouth *a* be closed by a rubber stopper, and the tap *f* connected by a rubber tube with a water-pump or other exhausting apparatus as the pressure inside diminishes, the rubber sheet bulges up inside the bell-jar, and pressing against the lower edge closes up any interstices due to irregularity of its own surface, or to imperfect grinding of the glass. A well-made water-pump will give within half an hour a high degree of exhaustion, and this without the use of any lubricant whatever. Where a very high degree of exhaustion is required, the application of a little grease outside round the lower edge of the bell-jar is advisable. The apparatus in this form can therefore be used for drying substances in vacuo, &c. For the purpose of rapid filtration such complete exhaustion is not, as a rule, required, and indeed is often positively detrimental, and defeats the object in view.

All the essential parts of the apparatus have been now described, everything else which is required for filtration being either in ordinary laboratory use, or else can be made with but little time and trouble, and at almost no cost.

When it is desired to collect the filtrate in a beaker or flask, the simplest arrangement is that represented in fig. 1. Fixed in *a* by means of a rubber stopper is an ordinary correct-angled funnel, fitted with platinum cone and filter in the ordinary way. To the stem of the funnel the glass tube *i*, bent as shown in the figure, and having the lower end ground obliquely, is attached by a short piece of black rubber tubing, the upper end of the tube being pushed up so as to be in actual contact with the stem of the funnel. The bend makes it easy to cause the end of the tube to touch the side of the beaker, which prevents spirting. It will be found convenient to keep several such tubes of different lengths ready made to suit larger and smaller vessels. Before commencing operations it is as well to slightly moisten the rubber sheet *d* with water. This is not by any means absolutely necessary, but causes a quicker gripping of bell-jar and rubber. The tap being in position No. 2, and connected with a water-pump in full action, filtering is commenced by first filling up the filter nearly full with the liquid to be filtered, and then establishing communication between the pump and the bell-jar by turning the tap to position No. 1. During the operation of transferring the precipitate to the filter, it is often necessary to lessen the rate of filtration by diminishing or destroying the difference of pressure outside and inside of the bell-jar. Instead of slipping off the rubber tubing connecting with the pump, this can be far more quickly and easily done by giving the tap a half-turn back to position No. 2, and thus allowing air to rush in both to the pump and to the bell-jar. The use of the knee-piece *h* will be now apparent, for by it the inrush of air is diverted away from the vessel inside, which might otherwise be blown against the side of the bell-jar, and upset or broken. As it is, however, the vessel inside is not at all affected by the inrush of air, however suddenly the tap be opened. In this connection another use of the knee-piece may be pointed out. Several of the otherwise very convenient and inexpensive high pressure water-pumps, now so much used, have a tendency to allow the water to run back under certain conditions, especially when a high degree of exhaustion is attained. Filtering directly into an exhausted flask, such an accident would cause the loss of an analysis, and a valve is therefore usually placed between the pump and the flask. Such a valve is quite unnecessary while using the above apparatus. The running back of the water can be at once stopped without diminishing the pressure inside the bell-jar by giving the tap a quarter turn, so as to admit air to the pump only. Even if water does run back into the bell-jar no harm is done, and by turning the knee-piece downwards the pump will suck back the water again almost to the last drop.

It is sometimes necessary in quantitative analysis to filter a small quantity of liquid directly into a small weighed platinum basin or crucible. For instance, in filtering the alkaline chlorides from the last traces of magnesia in a silicate analysis, or, to take another example, in the purification of minute quantities of alkaloids.

This may be readily done by the arrangement shown in fig. 3. The small funnel *l*, instead of fitting into the large stopper at *a*, fig. 1, is fixed by means of a small cork into the wide end of the stout glass tube *m* (fig. 3).

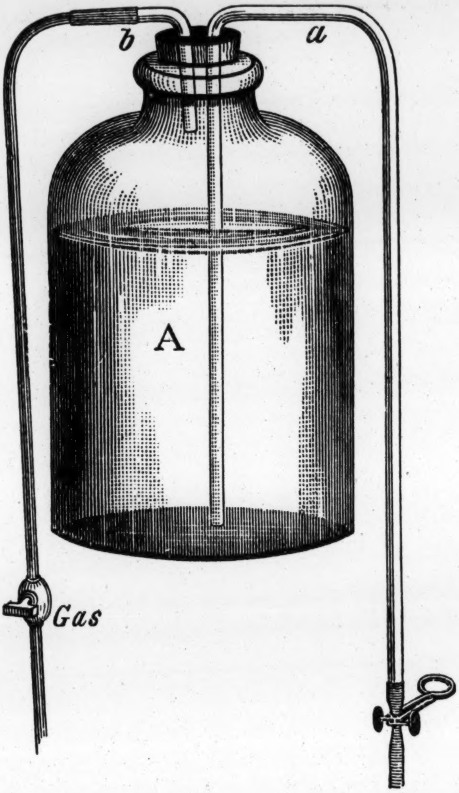
In order to avoid loss by spirting, the tube does not dip directly into the basin, but into the glass tube *n*. This tube may be made out of a broken pipette, and is supported by a loosely-fitting cork in the small bell-jar *o*, which may be most conveniently made by cutting off the lower half of an ordinary wide-mouthed bottle. When the filtration is over the small funnel *l* and cork should be removed, and the wide part of the tube *m* washed with a very little water, which serves at the same time to wash down the inside of the tube *n*. By this means the amount of wash water is reduced to a minimum. In the estimation of alkaloids the solvent is often chloroform or ether, which dissolve india-rubber, so that in such cases a small ordinary cork should be substituted for the rubber cork in *m*. It may be further pointed out that with this apparatus it is an easy matter to filter liquids containing hydrofluoric acid, which attack glass, by using a platinum funnel and filtering directly into a platinum basin or other platinum vessel.

A Convenient Method for Preserving Sulphuretted Hydrogen Water.

A large bottle A (fig. 4) is fitted with a double-bored cork. In the one hole is the syphon tube *a* leading from the bottom of the bottle to any convenient point lower than the other end of the tube, and closed at this lower end by a short piece of rubber tubing and an ordinary nipper tap. Through the other hole passes the short glass tube *b*, bent at right angles, and attached by means of rubber tubing to a piece of lead piping *c* fitted with a tap, below which tap *c* is soldered on to a pipe connected with the ordinary coal-gas supply.

By this arrangement the sulphuretted hydrogen is preserved from oxidation, and can always be run off perfectly clear. The gas tap should as a rule be kept closed, and need only be opened for an instant when the sulphuretted hydrogen water ceases to flow. It should be well oiled but not greased.

The only objection, if any, that I have been able to discover to this method is that after a time the smell of the sulphuretted hydrogen water becomes somewhat altered in character, apparently owing to the formation of traces of some organic sulphur compound, the nature of which I have not yet been able to examine.



2. On the Thermo-electric Position of pure Cobalt. By Professor Tait.

3. Transmission of Power by Alternate Currents. By Prof. George Forbes.

When a current of electricity is sent through a dynamo machine in the same direction as the current flows when the dynamo is being used as a generator, then the field magnets are polarised in their normal manner, and the magnetism of the armature is such as to cause it to rotate in the opposite direction to that in which it turns when generating a current. The dynamo now acts as a motor. If now the current be sent through the dynamo in the opposite direction, the field magnets are polarised in the opposite manner, *i.e.*, a north pole is found where a south pole was before. The same is true of the magnetism of the armature. Hence, in this case the poles, both of the field magnets and of the armature, are simply interchanged, and the rotation is again in the direction opposite to that in which it runs as a generator.* It thus appears that when a current of electricity is passed through an ordinary dynamo machine, the armature always rotates in one direction, whatever be the direction of the current. If, however, the field magnets or the armature be furnished with a current fixed in direction, the rotation of the armature may be reversed by reversing the direction of the current in the other part of the machine.

Such being the case, it would seem likely that if currents of varying direction be passed through a dynamo they would cause continuous rotation in one direction. I sent the current from a Siemens alternate-current machine through a small Griscom motor, which has a Siemens armature with only two reversals of the commutator in each revolution. Under these conditions the motor did not move at all, in whatever position of the armature it was tried. The reason of this is that the reversals of direction of the current are too rapid to allow of the complete reversal of the magnetism, both in the field magnets and the armature. This has been noticed before, and consequently it has been supposed that motive power cannot be transmitted by an alternate-current machine.

When the apparatus was in the same condition as described above, I gave a rapid rotation to the motor by hand, and after it had reached a certain speed the alternate-current maintained the rotation at a steady and uniform speed. I put some pressure on the spindle of the motor. It rotated at the same speed. I increased this pressure. The speed did not vary. But on continuing this action I arrived at a point where suddenly the driving action ceased, and if the motor continued to turn it was almost entirely owing to its own momentum; but not entirely so, for on removing the pressure entirely, while the motor was still running slowly, the speed rapidly increased until it reached the previous uniform rate. The sudden diminution of driving power, when a certain amount of friction was applied, was very remarkable.

What is the explanation of this action? The fact that a uniform speed of rotation is maintained, and never exceeded, points to a synchronism between the revolutions of the armature and the reversals of the alternate-current as being the true cause. In the Griscom motor the current comes from the source of electricity to one arm of the commutator attached to the Siemens armature, whence it is carried by the coil of wire surrounding the armature to the other arm of the commutator. The current leaving this arm of the commutator goes round the arms of the field magnets and magnetises the poles. Suppose now that by the rotation of the armature the reversal of the commutator takes place at the same time as the reversal of the current, then the two simultaneous reversals will cause the current always to circulate in the same direction through the coils, and one part of the armature will always be north and the other always south. But the current exciting the field magnets is constantly being reversed, although so rapidly as to allow of only a very slight magnetisation. Thus suppose that when the left side of the armature is north, the upper field magnet is north, it follows that when this one is south the other is south also, so that there is always repulsion and consequent rotation in a direction opposite to the hands of a watch.

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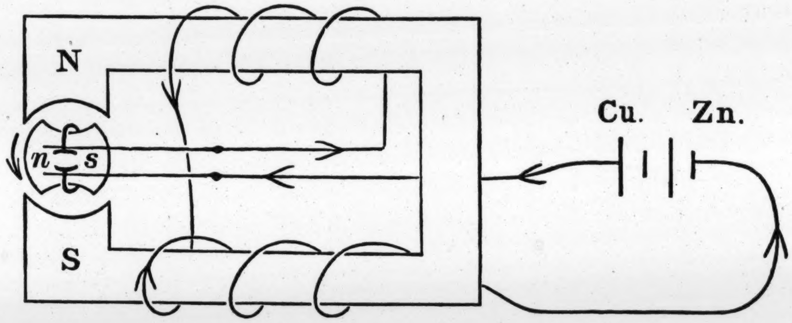
* I only speak of the magnetic actions for simplicity of expression. The electric actions are in the same direction as the magnetic.



It will be seen now that when we tried to start the motor with alternate currents, we had only the feeble interaction of the two pieces of iron extremely feebly magnetised; but when we have synchronous rotation, we have the more powerful interaction of a continuously excited and strong magnet (the armature), and the still feeble action of reversed magnetism of the field magnets. It is easy to see that if the armature tends to go too quickly or too slowly, the attractions between the movable and fixed parts will tend to check this irregularity. This accounts for the maintenance of the synchronism. It is also clear that if the speed be too much diminished by causing the armature to do work, the magnetic attraction will not act, and this amount of work will speedily stop the rotation of the armature. It is also clear that if the armature be *not* revolving in synchronism, and be doing no appreciable work, the magnetic attractions will always be tending to bring it more nearly to synchronism. These conclusions are all in accordance with what I observed.

This is all that I have done as yet in an experimental way on the transmission of power by alternate currents. It is easy to follow out the results which must follow when a machine with a Pacinotti collector (or commutator) is used, or when a magneto-electric motor is employed. But I will not enter on this subject until I have experimental data to support these conclusions.

I have considered these results, small as they may appear, to be worthy of the attention of this Society, because of the great value which I attach to the transmission of perfect synchronism. It is well known that in the construction of motors a great deal of ingenuity has been expended in endeavouring to obtain a steady speed, while the motor is doing a variable quantity of work. This difficulty is completely overcome by the use of alternate currents. It is true that the efficiency of my motor was small, owing to the feeble magnetism of the field magnets; but perhaps this can be overcome. I have said that the motor I used was one of the well-known Griscom motors, and the current I used was 1½ ampères, measured by a Siemens dynamometer. From an exceedingly rough estimate of the pressure on the pulley which was required to stop the motor, and the surface velocity of the pulley, I believe that I obtained about 200 ft. lbs. per minute.

But it is not the regularity of speed of motors which strikes me as important so much as the power to transmit absolute synchronism to a distance. It is, of course, well known that the main feature of many telegraph systems is "synchronous action." This is the case with the Hughes printing telegraph, so unfortunately neglected in this alone of all-important European countries. So it is with the Baudot and other modifications of the Hughes, and still more so is it the case with the autograph systems of Caselli, D'Arlingcourt, and others.

4. On the Homology of the Neural Gland in the Tunicata with the Hypophysis Cerebri. By W. A. Herdman, D.Sc., F.L.S., Professor of Natural History in University College, Liverpool.

(*Abstract.*)

In an ordinary simple Ascidian, where both the branchial and atrial apertures are at or near the anterior extremity of the body, the region lying between them—the interoscular area of Lacaze-Duthiers—is small in extent, and contains three important structures, the nerve ganglion, the neural gland, and the dorsal tubercle, lying close together. The ganglion in such a case is elongated dorso-ventrally, and gives off nerves at its two extremities, one set ventrally and anteriorly towards the branchial aperture, and the other set dorsally and posteriorly towards the atrial.

In those species, however, in which the atrial aperture is near or at the posterior end of the body, the interoscular area is large, and forms the dorsal edge of the body. The nerve ganglion may remain anterior in position, or it may be placed far back so as to be nearer to the atrial. In this case, the nerves given off from the extremities of the ganglion run, the one set anteriorly and the other posteriorly.

Wherever the ganglion may be, the neural gland is always found in close relation to it, either upon its posterior or ventral surface, according to the direction in which the long axis of the ganglion is placed. This neural gland consists of a mass of more or less ramified cæcal tubules imbedded in connective tissue, and all springing from a central space or wider tube which underlies the ganglion. The presence of this organ was first distinctly pointed out by Albany Hancock in 1868,* but this able investigator does not seem to have assigned any function to it.

The mysterious dorsal or "olfactory" tubercle was first described by Savigny in 1816, under the name of "tubercule anterieur." Since then it has been discussed by almost all who have worked at the Tunicata; it has received many names, but has usually been regarded as some sort of olfactory organ. It is invariably placed at the anterior end of the branchial sac, posterior to the circle of tentacles, and usually in a distinct, more or less triangular peritubercular area, which is a diverticulum of the prebranchial zone, and is formed by the dorsal ends of the peripharyngeal band bending posteriorly to meet the anterior extremity of the dorsal lamina.

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* "On the Anatomy and Physiology of the Tunicata," *Jour. Linn. Soc. Zool.*, vol. ix. p. 335.

The dorsal tubercle is, in the simplest form known, a funnel-shaped depression having its wider, circular, anterior end opening freely into the branchial siphon (the tube leading from the branchial aperture to the branchial sac), and separated off from the prebranchial zone by a raised edge or lip. The opposite narrower end is continued into a fine canal running dorsally or posteriorly in the direction of the nerve ganglion. This simple condition is found in *Molgula pedunculata;* in *Eugyra kerguelenensis,* the aperture is still wide, although its edge is square in place of being circular.

In other simple and compound Ascidians, the anterior half of the edge has been apparently pushed backwards so as to become invaginated and closely applied to the posterior half, thus reducing the circular aperture to a crescentic or semicircular slit. This condition is found in *Corella parallelogramma.* In most other forms, more or less complication is produced by the ends of the slit, or "horns" as they may be called, being prolonged often to a very great extent, and coiled in various directions, sometimes producing beautifully regular and closely placed spirals, as in *Molgula gigantea.* The patterns produced by this curving of the horns are very numerous, and often complicated; but their value in classification is slight, since they differ sometimes to a considerable extent in different individuals of the same species, and on the other hand, are sometimes very similar in members of different genera or even families.

Some few forms are known in which, in place of being a single curved or spiral organ, it is formed of several distinct irregularly-shaped apertures, as in *Cynthia irregularis;* this has evidently been formed by the lips of the primitive simple dorsal tubercle having become so folded as to divide the single aperture into several. Lastly, there are cases in which the organ is even more complicated, as in *Boltenia pachydermatina,* so that it becomes very difficult to trace its derivation from a simple circular opening.

This variously shaped organ is histologically merely a depression in the connective tissue of the mantle, lined by epithelium continuous with the squamous epithelium covering the prebranchial zone. These cells are modified upon the edges of the slit into first cubical, and then columnar ciliated cells. The cilia project into the cavity of the organ.

Since the time of Savigny, the dorsal tubercle has been almost universally regarded as a sense organ of some kind—probably olfactory or gustatory, or in some way capable, as Hancock * suggested, of "testing the quality of the inhaled water." The reasons for this view have been—

1. The position of the organ at the entrance of the branchial sac, where a sense organ would be of great apparent value.

2. Its structure—a ciliated depression covered in part by columnar cells, some of which closely resemble sense cells.

3. Its intimate relation with the ganglion, and the presence of a nerve arising from the anterior end of the ganglion, running towards the branchial aperture, close past the dorsal side of the tubercle, and presumably supplying it with nerves.

In 1876, Ussow † showed that the neural gland lying below the ganglion was continued into a delicate duct, lined by cubical epithelium, which ran forwards and opened into the tubular posterior end of the funnel-like depression forming the dorsal tubercle, so that the variously-shaped slit of the tubercle was thus shown to be merely the aperture of the duct from the neural gland. Long previous to this, in 1861, Keferstein and Ehlers ‡ had shown that the funnel-shaped cavity forming the dorsal tubercle in *Doliolum denticulatum* was continued backwards as a delicate duct to the base of the ganglion, but they regarded this duct and its anterior expansion as a prolongation from the ganglion itself.

In 1881, Ch. Julin § confirmed Ussow's discovery, and described minutely the condition of the gland, the duct, and the tubercle in several species of simple Ascidians. He also declared that there was no connection between the nerve running from the ganglion to the branchial aperture and the tubercle, and that consequently the latter could not be a sense organ, and was nothing more than the opening of the duct. In a second paper published shortly afterwards, Julin || described the condition of these organs in two

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* *Loc. cit.* p. 335.

† *Proc. Imp. Soc. Nat. Hist., &c., Moscow*, vol. xviii. fasc. ii. (in Russian).

‡ *Zoologische Beiträge*, iii., "Ueber die Anatomie und Entwicklung von Doliolum," p. 61, Leipzig.

§ *Archives de Biologie*, vol. ii. p. 59.

|| *Archives de Biologie*, vol. ii. p. 211.

additional species, and enunciated the theory suggested to him by E. van Beneden, that the neural gland was renal in function, and was the homologue of the hypophysis cerebri of the vertebrate brain. In favour of this homology may be considered—

1. The position of the gland upon the ventral surface of the nerve centre and above the pharynx.

2. Its glandular nature.

3. Its connection with the anterior end of the pharynx by a duct—Balfour, Kölliker, and others having shown that the hypophysis or pituitary gland in higher vertebrates arises as a dorsal diverticulum from the stomodæum, but afterwards loses this connection.

From my own observations on a number of different forms of the Tunicata, I can confirm Ussow and Julin's statements as to the presence of a duct from the neural gland and its connection with the slit of the dorsal tubercle, and, like Julin, I am unable to find any nerve supplying the supposed sense organ. I have, however, in several cases seen certain of the epithelial cells covering the edges of the slit which had a striking resemblance to sense cells, such as those in the ectoderm of *Actiniæ.* This observation taken along with Julin's descriptions, and especially with the condition of affairs in some specimens of *Ascidia mammillata* which I have recently examined, has suggested to me that possibly the dorsal tubercle may be *both* the aperture of a gland corresponding to the hypophysis cerebri, and also a sense organ, probably of an olfactory or gustatory nature.

Ascidia mammillata is one of the forms discussed by Julin in his second paper. It is a large species, with the branchial and atrial apertures rather far apart, and the ganglion at a considerable distance from the anterior end of the body, the interoscular area being of large extent. Julin found that the neural gland in this species did not form the usual compact mass, but was in a somewhat rudimentary condition, and that besides having the usual duct running anteriorly to communicate with the pharynx by the dorsal tubercle, it had also a number of short funnel-shaped apertures into the peribranchial or atrial cavity enclosed by the mantle; so that in this species the products of the neural gland might be excreted either into the branchial sac by the main duct and dorsal tubercle, or into the dorsal part of the peribranchial cavity by the lateral shorter ducts This peribranchial cavity communicates directly with the exterior by means of the atrial aperture, and is the cloacal cavity into which the rectum and the genital ducts open. Its lining membrane is derived from the epiblast, being formed in the embryo by a pair of lateral involutions which afterwards fuse dorsally.

In two specimens of *Ascidia mammillata* which I had an opportunity of examining recently, I found the neural gland in precisely the condition described by Julin, but its duct had no aperture into the pharynx, the dorsal tubercle being entirely absent. The small funnel-shaped apertures into the peribranchial cavity were numerous and well developed, so that in the case of these individuals the neural gland was connected with the cloacal part of the peribranchial cavity only, exactly the arrangement to be expected if the gland had a renal function. Admitting that that is so, and that the neural gland is the homologue of the hypophysis, it seems possible to me that this, or something like this, may have been the condition of affairs in the primitive Chordata previous to the point of divergence of the Urochorda.* There may have been a renal gland placed ventrally to the nervous system—not necessarily at the anterior end only,—and opening on the surface of the body † by one or more laterally placed apertures—this gland being represented in the Tunicata by the neural gland, and in the Vertebrata by the glandular portion of the pituitary body.

As to the dorsal tubercle, whether it has now any sensory function is doubtful, on account of the apparent absence of nerve-supply; but I consider that it must have been a sense organ formerly—possibly placed at first on the surface of the body close to the mouth (the branchial aperture), since the anterior part of the pharynx develops from the epiblast as a stomodæum,—and I would suggest that the connection of the tubercle with the duct of the neural gland may be an afterchange, caused possibly by the enlargement of the pharynx into a branchial sac, and the development of the peribranchial chamber. It may readily be imagined how, as the result of the formation of these cavities, the dorsal tubercle would be brought into closer relation with the neural gland, and one or more of the funnel-shaped ducts of the gland might, after having been carried in from the surface by the formation of the lateral atrial involutions, come to open into the ciliated depression of the tubercle in place of into the peribranchial cavity. The condition thus produced would be very much what Julin has described as existing in his specimens of *Ascidia mammillata.* If now for some reason the original openings into the peribranchial cavity became suppressed, leaving merely the secondary opening into the pharynx by means of the dorsal tubercle, we would arrive at the condition found in all ordinary Ascidians. It is not easy to see what the cause of the change could be, as there is no apparent advantage to be derived from it; probably, however, there is no disadvantage since there is abundant communication between the branchial sac and the peribranchial cavity through the stigmata or slits in the wall of the former.

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* I have adopted Balfour's classification (*Comparative Embryology*, vol. ii. 1881) as follows:—

CHORDATA.

1. Urochorda (Tunicata).
2. Cephalochorda (*Amphioxus*).
3. Vertebrata.

† The lining of the peribranchial cavity into which the ducts open in *Ascidia mammillata* has been already shown to be continuous with the ectoderm on the surface of the body.

This suggestion as to the origin of the present structure and relations of the neural gland and neighbouring organs, in most Tunicata, implies that the pituitary body in the Vertebrata, which has lost its connection with the exterior, and probably also its function, has a similar history. In this view I am encouraged by some remarks by Balfour * from which it is clear that he considered the pituitary body, judging from its development, to have been originally a sense organ, opening into the mouth, and possibly corresponding to the Ascidian dorsal tubercle. He has also suggested † as an alternative the possibility that the neural gland in the Tunicata may be the homologue of the vertebrate pituitary body. This is of course the theory supported by van Beneden and Julin, and is open to the objection that it does not account for the remarkable structure of the dorsal tubercle. The view I hold combines both of those suggested by Balfour and Julin, by considering the pituitary body as the homologue of the neural gland, and as being therefore the rudiment of a primitive neural organ ‡ which opened in the early Chordata by lateral ducts upon the side wall of the body; while the connection of the pituitary body with the stomodæum, in embryo vertebrates, is regarded as being not its original and proper duct, but a secondary connection which has been formed with a lost sense organ, placed at or in front of the anterior end of the pharynx, and homologous with the dorsal tubercle in the Tunicata.

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* *A Treatise on Comparative Embryology*, vol. ii. p. 359, London, 1881.

† *Loc. cit.*, p. 360.

‡ Not the pronephros, since that is found along with the pituitary body in many Vertebrates, but possibly more ancestral. Might it not be the homologue of the provisional trochosphere excretory organs described by Hatschek and others in *Polygordius* and some Mollusca?

In conclusion, Ussow and Julin have conclusively shown that the dorsal tubercle is not *merely* a sense organ. The complex structure which the tubercle usually presents seems to indicate that it is not *merely* the aperture of a duct. Whether, as I suggest, it may be a sense organ into which the duct has come to open can scarcely be determined on the evidence at present in our hands. The lines of investigation which may be reasonably expected to throw additional light upon the matter are—(1) The exact course of development of the neural gland and the dorsal tubercle, and further information as to the pituitary body; and (2) The examination of the condition of the gland and its ducts throughout the Tunicata, and especially in a large number of specimens of *Ascidia mammillata*, a species in which these organs appear to be in a variable and highly interesting condition.

5. On the Quaternion Expression of the *Finite* Displacements of a System of Points of which the Mutual Distances remain Invariable. By Gustave Plarr, Docteur ès-Sciences. Communicated by Professor Tait.

When we try to define, from a purely geometrical point of view, the *rigidity* of a system of points, we are inclined to consider the invariability of the mutual distances of the points a sufficient definition. But when we take this definition as a starting point, in the problem of expressing the *finite* displacements of the system, we soon discover that the invariability of the distances is not sufficient for insuring the invariability of the relative positions of the points.

In this paper we have, with the help of the quaternion-method, transformed the fundamental condition of the problem into an equation which presents two separate factors. One of the factors solves the problem by representing the displacements as a *screw-rotation* of the system. The other factor gives the geometrical *perversion* of the figure which the system possessed before the displacement of its points.

From the expressions of the two kinds of displacements, we are enabled to deduce the supplementary condition by which the rigidity of a geometrical system shall be established in all its completeness.

Both these expressions are reducible to the same definite form, each expression yielding properties which correspond each to the other in the two kinds of displacement. To the *line* representing the axis of rotation in the screw-rotation corresponds the *plane* representing the plane of symmetry in the perversion; to the translation in the first, parallel to the definite axis of rotation, corresponds a translation in the second, parallel to the definite plane of symmetry.

With the expressions combined of the two kinds of displacement we shall be able to establish a general demonstration of the proposition according to which two given successive perversions are equivalent to a determinate screw-rotation.

§ 1.

Let us consider first the displacements of two points A and B, supposing that they displace themselves to A′ and B′ respectively.

Assuming an arbitrary origin O_1 for the vectors of these points, we put

$$O_1A = \alpha, \qquad O_1B = \beta,$$
$$O_1A' = \alpha', \qquad O_1B' = \beta'.$$

By our hypothesis we must have

$$(1) \qquad \begin{cases} T\alpha' = T\alpha, \\ T\beta' = T\beta, \\ T(\beta' - \alpha') = T(\beta - \alpha). \end{cases}$$

We transform the third of these equations, by the help of the two first, into

$$(2) \qquad S\alpha'\beta' - S\alpha\beta = 0.$$

The most general solution of the two first equations may receive the form

$$(3) \qquad \begin{cases} \alpha' = p\alpha p^{-1}, \\ \beta' = q\beta q^{-1}, \end{cases}$$

where p and q may be any versors whatever.

By these expressions the first member of (2) becomes

$$Sp\alpha p^{-1}q\beta q^{-1} - S\alpha\beta.$$

Let us call Z this expression, so that

$$Z = 0$$

will represent the equation (2).

We represent $q^{-1}p$ by a single letter r, so that

$$q^{-1}p = r\,,$$

giving

$$p^{-1}q = r^{-1}\,.$$

Thus we get

$$\mathrm{Z} = \mathrm{S}\beta(r\alpha r^{-1} - \alpha)\,.$$

Having generally

$$(5) \qquad r\alpha r^{-1} - \alpha = r(\alpha r^{-1} - r^{-1}\alpha) = 2r\mathrm{V}.\,(\mathrm{V}r\ \ \alpha)\,,$$

we get

$$\mathrm{Z} = 2\mathrm{S}.\,[\beta\,.\,r\mathrm{V}(\mathrm{V}r.\,\alpha)]\,.$$

Let ζ be the unit vector of $\mathrm{V}r$, and let us introduce the factor

$$\zeta^2 = -1$$

under the sign S. Then we get

$$\mathrm{Z} = -2\mathrm{S}.\,[\zeta\,.\,\zeta\beta\,.\,r\,.\,\mathrm{V}(\mathrm{V}r\alpha)]\,.$$

Of course, we see at a glance that Z will vanish when we assume

$$\mathrm{TV}r = 0\,,$$

in which case we get $r = \pm 1$, and consequently $\pm q = p$: this will be the first solution spoken of. But we will continue to examine the expression of Z independently of the vanishing of $\mathrm{V}r$.

Decomposing $\zeta\beta$ into its scalar and its vector, we notice that the term in Z depending on $\mathrm{S}\zeta\beta$ will vanish, because the factor of $\mathrm{S}\zeta\beta$ will have a vector under the sign S. We have therefore

$$\mathrm{Z} = -2(\mathrm{TV}r)\mathrm{S}[\zeta\mathrm{V}\zeta\beta\,.\,r\,.\,\mathrm{V}\zeta\alpha]\,,$$

or replacing r by $q^{-1}p$, and grouping the factors, we get

$$(6) \qquad \mathrm{Z} = -2(\mathrm{TV}q^{-}p)\mathrm{S}[\zeta(\mathrm{V}\zeta\beta\,.\,q^{-1})(p\mathrm{V}\zeta\alpha)].$$

Thus far we could advance without making any particular hypothesis about the versors p and q. Now we see that if both had the same axis, that common axis would be identical with ζ, and we could at once deduce some further transformation.

Let us examine how far the generality of the solution of $\mathrm{Z} = 0$ will be restricted if we introduce the hypothesis that p and q be co-axial, namely, $\mathrm{UV}p = \mathrm{UV}q$?

Considering this incidental question from a purely analytical point of view, we observe that the six elements comprised in α' and β' are by the two first conditions (1) reduced to four independent elements, α and β being given. By (3) we introduce six elements contained in p and q taken together, and, if we introduce the condition

$$(7) \qquad \mathrm{UV}p = \mathrm{UV}q = \zeta,$$

we reduce to four the number of the independent elements contained in p and q. By this means we do not reduce the number of the independent elements on which α' and β' depend.

Considering the question in its geometrical aspect, we represent the finite displacements by $d\alpha$ and $d\beta$,* putting

$$(8) \qquad \alpha' = \alpha = d\alpha, \quad \beta' = \beta = d\beta,$$

and applying (5) to the expressions (3), we get

$$(9) \qquad d\alpha = 2p\mathrm{V}(p \,.\, \alpha), \quad d\beta = 2q\mathrm{V}(\mathrm{V}q \,.\, \beta).$$

Hence

$$(10) \qquad \mathrm{S} \,.\, d\alpha\mathrm{V}p = 0, \quad \mathrm{S}.\, \delta\beta\mathrm{V}q = 0.$$

The directions of $\mathrm{V}p$ and $\mathrm{V}q$ are thus limited to planes respectively perpendicular to $d\alpha$ and to $d\beta$, these planes passing through the origin O_1 of α and β. Moreover these planes contain a definite point each. Namely, if we put (1) under the form

$$(\alpha + d\alpha)^2 - \alpha^2 = 0,$$

we deduce

$$\mathrm{S}(\alpha + \tfrac{1}{2}d\alpha)d\alpha = 0\,;$$

likewise we get

$$\mathrm{S}(\beta + \tfrac{1}{2}d\beta)d\beta = 0.$$

If then α and β, $d\alpha$ and $d\beta$ be given, the axis of p and q may still remain of an indeterminate position each in its plane, perpendicular to $d\alpha$, and containing $\alpha + \frac{1}{2}d\alpha$, or perpendicular to $d\beta$, and containing the point $\beta + \frac{1}{2}d\beta$.

But these two planes generally intersect each other, and if ζ represents the unit-vector in the direction of the line of intersection, we may take this direction for those of $\mathrm{V}p$ and $\mathrm{V}q$ in common without affecting the values of $d\alpha$ and $d\beta$.

We define therefore the expressions of p, q by

$$(11) \qquad \begin{cases} p = \cos u + \zeta \sin u, \\ q = \cos v + \zeta \sin v, \end{cases}$$

and we determine the angles u and v by the relations (9); inversely if we give ourselves ζ, u, v as data, the expressions (9) will represent any values of $d\alpha$ and $d\beta$ compatible with the two first conditions (1).

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* The characteristic d representing finite differences throughout this paper.

Should $d\alpha$ and $d\beta$ require to be parallel, then we would, *à fortiori*, be entitled to give to Vp and to Vq the same direction, considering that both would be comprised in one and the same plane, and the relations (9) would again determine u and v.

Let us now introduce the expressions (11) into Z, remembering that we have now by (9)

$$(12) \qquad d\alpha = 2pV\zeta\alpha \,.\, \sin u, \quad d\beta = 2qV\zeta\beta \sin v \,.$$

Moreover, as

$$S \,.\, qV\zeta\beta = 0 \,,$$

we may write

$$V\zeta\beta \,.\, q^{-1} = qV\zeta\beta \,.$$

We have now by (12)

$$Z = \frac{TV(q^{-1}p)}{2 \sin u \sin v} S \,.\, \zeta d\alpha d\beta \,.$$

The relations (10) become

$$(13) \qquad S\zeta d\alpha = 0 \,, \quad S\zeta d\beta = 0 \,.$$

This gives

$$\zeta = \pm UV \,.\, d\alpha d\beta$$
$$S\zeta d\alpha d\beta = \mp TV.d\alpha d\beta \,.$$

The equation $Z = 0$ becomes then finally

$$\frac{T.V(q^{-1}p)}{2 \sin u \sin v} T.V(d\alpha d\beta) = 0 \,.$$

We have therefore two solutions for the equation (2),

I. either $\qquad T.V(q^{-1}p) = 0$, namely $\pm q = p$;
II. or $\qquad TV(d\alpha d\beta) = 0$,

namely, $d\alpha$ and $d\beta$ to be parallel to one another.

We omit such particular cases for which both conditions are satisfied at the same time.

§ 2.

For the discussion of these two solutions we will refer to a fixed origin O, the position of the points which are invariably linked to the system. Let ρ_1, ρ_2, &c., ρ, designate the vectors of these points before their displacements, and let ρ'_1, ρ'_2, &c., ρ' be the vectors of respectively the same points after their finite displacement. For the sake of conciseness, we may designate the points themselves by their vectors. We will also assume that the vectors ρ_1, ρ_2, &c., be given: the vectors ρ'_1, ρ'_2, &c., ρ' will be deduced from them according to the one or the other of the two solutions in question.

Let us consider the groups of three points, ρ_1, ρ_2, ρ, where ρ is to represent (successively) any point of the system, save ρ_1 and ρ_2. We put

$$\rho_2-\rho_1=\alpha .$$

And we liken $\rho-\rho_1$ with β, putting

$$\rho-\rho_1=\beta .$$

The origin of α and β will be at the extremity of ρ_1, say in O_1.

Introducing, for any index, the notation

$$\rho'=\rho+d\rho ,$$

we shall have now

$$d\alpha=d\rho_2-d\rho_1 ,$$
$$d\beta=d\rho-d\rho_1 .$$

We assume $d\rho_1$ to be given, and then the object of each of the two solutions will be the determination of $d\rho_2$, $d\rho_3$, &c. $d\rho$.

The first solution,

$$TV(q^{-1}p)=0 ,$$

gives $q^{-1}p=\pm 1$, or $q=\pm p$. But the double sign disappears in the operator $q\ (\quad)q^{-1}$, which becomes $p\ (\quad)p^{-1}$. We have therefore the same operator for all the displacements. Thus we get

$$(14) \qquad \rho'_2-\rho'_1=p(\rho_2-\rho_1)p^{-1},$$

$$(15) \qquad \rho'-\rho'_1=p(\rho-\rho_1)p^{-1}.$$

Writing (15) for $\rho=\rho_3$, we have

$$(16) \qquad \rho'_3-\rho'_1=p(\rho_3-\rho_1)p^{-1}.$$

The relations (16) and (14) establish the invariability of the triangle, which has its summity in ρ_1, ρ_2, ρ_3, before the displacement, and, in ρ'_1, ρ'_2, ρ'_3 after the displacement of the system.

Looking on this triangle as a basis in reference to which the position of any other point of the system may be defined, we get by (15), and by subtracting successively (14) and (16) from (15),

$$(17)\qquad \begin{cases} \rho' - \rho'_1 = p(\rho - \rho_1)p^{-1}, \\ \rho' - \rho'_2 = p(\rho - \rho_2)p^{-1}, \\ \rho' - \rho'_3 = p(\rho - \rho_3)p^{-1}, \end{cases}$$

and by these relations the distances of ρ' from the summits $\rho'_1, \rho'_2, \rho'_3$ of the displaced triangle are equal to the corresponding distances of ρ from the summits ρ_1, ρ_2, ρ_3 before the displacement of the system.

If we take the scalar of the product member to member, we get, owing to $p^{-1}p = 1$:

$$(18)\qquad S(\rho' - \rho'_1)(\rho' - \rho'_2)(\rho' - \rho'_3) = +S(\rho - \rho_1)(\rho - \rho_2)(\rho - \rho_3).$$

We shall refer again to this result when we have the corresponding result calculated by the second solution.

By the second solution, we must satisfy the condition

$$TV(d\rho_2 - d\rho_1)(d\rho - d\rho_1) = 0,$$

and as ρ represents any point of the system, it follows that all the relative displacements $d\rho - d\rho_1$ must be parallel to one another.

Let us designate by η the unit-vector in the direction of these displacements. We shall then have, when $\beta = \rho - \rho_1$,

$$d\beta = \eta g$$
$$\beta' = \beta + \eta g$$
$$\beta'^2 = \beta^2 + 2gS\eta\beta - g^2.$$

Applying (1), namely $T\beta' = T\beta$, we get

$$0 = g(2S\eta\beta - g).$$

Omitting $g = 0$, which corresponds to no displacement at all, we get

$$d\beta = 2\eta S\eta\beta,$$

and

$$(19)\qquad \beta' = \beta + 2\eta S\eta\beta;$$

or with the signification of β:

$$\rho' - \rho'_1 = \rho - \rho_1 + 2\eta S\eta(\rho - \rho_1).$$

From this, making successively $\rho = \rho_2$, $\rho = \rho_3$, and proceeding as in the case of the first solution, we establish, first the invariability of the triangle of which the summits are in ρ_1, ρ_2, ρ_3, before the displacement, and in $\rho'_1, \rho'_2, \rho'_3$ after the displacement. Secondly, we establish the three relations following:

$$(20)\qquad \begin{cases} \rho' - \rho'_1 = \rho - \rho_1 + 2\eta S\eta(\rho - \rho_1), \\ \rho' - \rho'_2 = \rho - \rho_2 + 2\eta S\eta(\rho - \rho_2), \\ \rho' - \rho'_3 = \rho - \rho_3 + 2\eta S\eta(\rho - \rho_3), \end{cases}$$

by which the distances of ρ' from ρ'_1, ρ'_2, ρ'_3 are respectively the same as those of ρ from ρ_1, ρ_2, ρ_3.

We now take the scalar of the product member to member. This gives for the second member,

$$\begin{gathered} S.(\rho - \rho_1)(\rho - \rho_2)(\rho - \rho_3) \\ + 2S.\eta\Sigma V(\rho - \rho_1)(\rho - \rho_2)\ S\eta(\rho - \rho_3), \end{gathered}$$

where Σ represents the summing of the three terms obtained by the circular permutation of the indices. But this sum represents the vector

$$\eta S.(\rho - \rho_1)(\rho - \rho_2)(\rho - \rho_3),$$

and, treated by $2S\eta$, we get a term which reduces itself so as to give:

$$(21)\qquad S.(\rho' - \rho'_1)(\rho' - \rho'_2)(\rho' - \rho'_3) = -S.(\rho - \rho_1)(\rho - \rho_2)(\rho - \rho_3).$$

If we compare the sign in the second member of this result (21) with the sign in the corresponding result (18), we see at a glance that the displacements of the second solution produce a state of things which is incompatible with the conditions which constitute a physically rigid system, and that therefore the relation (18) alone characterises the systems which possess the complete rigidity from a geometrical point of view.

As we have at our disposition *two* different expressions of the displacements according to the second solution, namely, the expressions (3) (with the values (11) of p and q) and the general expression (19), we will investigate into the properties of the second solution in using both expressions. But we will discuss first the properties of the first solution, and then, taking up the second, we will at the same time be able to show the correspondence (from a formal point of view) of the properties of both solutions.

§ 3.

By the first solution we have the expression (17), of which we write again the first

$$(22)\qquad \rho' - \rho'_1 = p(\rho - \rho_1)p^{-1},$$

and which applies the same operator $p\,(\quad)\,p^{-1}$ to any point of the system. Let us look upon p as a datum. Moreover as

$$\rho'_1 = \rho_1 + d\rho_1\,,$$
$$\rho' = \rho + d\rho\,,$$

we assume also that the displacement $d\rho_1$ of the extremity of ρ_1 be *given*, so that the expression (22) will serve to the determination of the displacement $d\rho$ of the extremity of ρ, which may be any point of the system, or also, in this case any point invariably connected with the points of the system of given points.

Let us determine the point, or locus of points, of which the displacement is parallel to the axis ζ of p. If ρ_0 designates such a point we define it by putting

(23) $$\mathrm{V}.\zeta d\rho_0 = 0\,,$$

or

(23 *bis*) $$d\rho_0 = \zeta t\,,$$

where t is a scalar to be determined.

Applying the first of (17) we get for $\rho = \rho_0$:

(24) $$\rho_0 + \zeta t - (\rho_1 + d\rho_1) = p(\rho_0 - \rho_1)p^{-1}\,.$$

Transforming this by (5) and multiplying by p^{-1}, we get

$$\frac{1}{2\sin u}\,p^{-1}(\zeta t - d\rho_1) = \mathrm{V}\zeta(\rho_0 - \rho_1)\,.$$

By this equation $\mathrm{S}\zeta(\rho_0 - \rho_1)$ remains indeterminate. We represent this scalar by $-z$ and add member to member

$$-z = \mathrm{S}\zeta(\rho_0 - \rho_1)\,.$$

Multiplying the sum by $-\zeta$ we get

$$\zeta z + \frac{p^{-1}}{2\sin u}(-\zeta^2 t + \zeta d\rho_1) = \rho_0 - \rho_1\,.$$

Taking first the scalar of both members gives

(25) $$t + \mathrm{S}\zeta d\rho_1 = 0\,,$$

so that t is determined by this relation, and there remains

(26) $$\rho_0 = \zeta z + \left[\rho_1 + \frac{p^{-1}}{2\sin u}\,\mathrm{V}.\zeta d\rho_1\right].$$

By this expression we see that ρ_0 represents the vector of any point of a straight parallel to ζ (z being an indeterminate scalar), and passing through a definite point, of which the vector depends on p and on ρ_1 and $d\rho_1$, all the elements of which are given, or supposed to be given.

Subtracting (24) member to member from (22) in which we replace ρ' by $\rho + d\rho$, and ρ'_1 by $\rho_1 + d\rho_1$, we get

$$(27) \qquad d\rho = \zeta t + d\sigma\,,$$

where we put

$$(28) \qquad d\sigma = [p(\rho - \rho_0)p^{-1} - (\rho - \rho_0)]\,.$$

The displacement $d\rho$ is therefore represented by two components. The first, which is parallel to ζ, is the same for all the points, because (27) gives, by $S\zeta d\sigma = 0$,

$$S\zeta d\rho = -t\,,$$

and, by (25), the value of t depends on the data alone, and is therefore a constant. In fact, by this last relation we have the series of equations

$$(29) \qquad -t = S\zeta d\rho_1 = S\zeta d\rho_2 = S\zeta d\rho_3 = \&c.,$$

which show that the displacements of all the points of the system, when projected on the direction ζ give one and the same projection, so that we may continue to designate ζt by $d\rho_0$ as by (23 *bis*).

The component $d\sigma$ is perpendicular to ζ, because, owing to $\zeta p = p\zeta$, we have

$$S\zeta p(\rho - \rho_0)p^{-1} = Sp^{-1}p\zeta(\rho - \rho_0) = S\zeta(\rho - \rho_0)\,.$$

Hence,

$$S\,.\,\zeta d\sigma = 0\,.$$

This relation assigns to $d\sigma$ a plane perpendicular to ζ. As to the direction of $d\sigma$ within that plane, we must deduce it from (28). By its definition, $d\sigma$ represents the displacement of $\rho - \rho_0$ after a conical rotation round the axis parallel to ζ, which passes through the point ρ_0, and which therefore is the straight line (26) representing the locus of ρ_0; the angle of rotation being $2u$, namely, twice the angle of p, when

$$p = \cos u + \zeta \sin u\,.$$

We may transform $d\sigma$ in many ways. As, for example, putting

$$\rho - \rho_0 = \sigma\,,$$

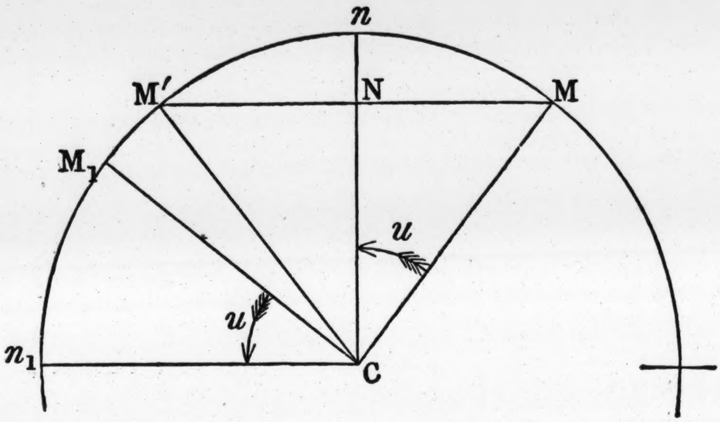
we have

$$(30) \qquad d\sigma = p\sigma p^{-1} - \sigma\,.$$

Applying (5), we get

$$(31) \qquad d\sigma = 2 \sin u\,.\,pV\zeta\sigma\,.$$

If $\overline{\mathrm{CM}}$ represents the projection of σ on a plane perpendicular to ζ (and we assume ζ to be perpendicular to the paper and the positive part of ζ above it), then $\overline{\mathrm{CM}_1}$ perpendicular to $\overline{\mathrm{CM}}$, M_1 being on the circumference of the circle of radius CM, and to the left of CM, will represent $\mathrm{V}.\zeta\sigma$. Taking $\mathrm{M}_1\mathrm{C}n_1 = u$, n_1 being on the circumference, we get



$$\overline{\mathrm{C}n_1} = p\mathrm{V}.\zeta\sigma\,.$$

This gives the direction of $d\sigma$. We then draw the chord $\overline{\mathrm{MM}'}$ parallel to $\overline{\mathrm{C}n_1}$, and CN perpendicular to the chord, so that the angles

$$\mathrm{MCN} = \mathrm{M}_1\mathrm{C}n_1 = u\,,$$

and therefore the length $\overline{\mathrm{MN}} = \mathrm{C}n_1 \times \sin u$. Hence we have

$$d\sigma = 2\overline{\mathrm{MN}} = \overline{\mathrm{MM}'}.$$

This construction is more laborious than the one which represents $d\sigma$ when expressed by

$$(32) \qquad d\sigma = (p^2 - 1)\mathrm{V}(\mathrm{V}\zeta\sigma\,.\,\zeta)\,,$$

where $\mathrm{V}(\mathrm{V}\zeta\sigma\,.\,\zeta)$ represents $\overline{\mathrm{CM}}$, the projection of σ on the plane perpendicular to ζ, and where $p^2\mathrm{V}.(\mathrm{V}\zeta\sigma\,.\,\zeta)$ represents $\overline{\mathrm{CM}}'$, namely, the result of the rotation of $\overline{\mathrm{CM}}$ to the amount of twice the angle of p round the axis ζ.

The two components of $d\rho$ produce, as it is now seen, a displacement which would take place if the system was invariably connected with a screw which, in moving in its nut, displaces itself along the axis of the screw to a distance ζt, while the screw turns round its axis by an angle $2u$. The pitch h of the screw being $\frac{t}{2u}=h$.

The displacement, according to the first solution, is therefore appropriately called a *screw-rotation.*

When the angle of rotation is infinitely small, then the versor p in the expression (31) reduces itself to unity, and if we designate by ϵ the instantaneous axis of rotation $\zeta \times 2u$, we get by (27) and (31)

$$(33) \qquad d\rho = \epsilon h + \mathrm{V}\epsilon(\rho - \rho_0).$$

In the expression (27) of the *finite* displacement $d\rho$, we may consider now the *data* to be the *six* following scalar elements :—

The *two* which determine ζ; the *two* which determine the position of the axis of the screw by the point where it intersects a plane perpendicular to ζ passing through the origin O of the vectors ρ, this point being

$$\mathrm{V}(\mathrm{V}\zeta\rho_0 . \zeta) \text{ or } \mathrm{V}.(\zeta\mathrm{V}.\rho_0\zeta),$$

(the part $-\zeta\mathrm{S}\zeta\rho_0$ disappearing in the expression of $d\rho$); and, finally, the angle u and the value of t, forming the last *two* elements.

Likewise the expression (33) depends on the three scalar elements contained in ϵ, on the two which determine $\mathrm{V}(\mathrm{V}\zeta\rho_0 . \zeta)$, and finally on h, six elements in all.

§ 4.

For the discussion of the second solution we will write the first of the expressions (20) under the form

$$(34) \qquad d\rho - d\rho_1 = 2\eta\mathrm{S}\eta(\rho - \rho_1).$$

We will look upon $d\rho_1$ and η as the principal *data* by which to determine the displacement $d\rho$ of any point ρ, when of course ρ_1 and ρ are given.

Let us determine the point ρ_e, or locus of points, for which the displacement is perpendicular to η. The condition will be (analogous to (23)) for $\rho = \rho_e$,

$$(35) \qquad \mathrm{S}\eta d\rho_e = 0.$$

Writing (34) for $\rho = \rho_e$, we get

$$(36) \qquad d\rho_e - d\rho_1 = 2\eta\mathrm{S}\eta(\rho_e - \rho_1).$$

Treating this by $\mathrm{S}.\eta$ we get, by (35),

$$(37) \qquad -\mathrm{S}\eta d\rho_1 = -2\mathrm{S}\eta(\rho_e - \rho_1);$$

or if we represent by C the known quantity,

$$C = -S\eta(\rho_1 + \tfrac{1}{2}d\rho_1),$$

we get for the locus of ρ_e,

$$(38) \qquad S\eta\rho_e + C = 0.$$

This represents a plane perpendicular to η, its distance from the origin O of the vectors ρ being $= \eta C$.

The value of $d\rho_e$ becomes, by (36) and (37),

$$d\rho_e = d\rho_1 + \eta S\eta d\rho_1,$$

or,

$$d\rho_e = V(V\eta d\rho_1 . \eta),$$

namely, a vector perpendicular to η, as we intended it to be, and known, η and $d\rho_1$ being given.

If we subtract (36) member to member from (34) we get now

$$d\rho = d\rho_e + d\tau,$$

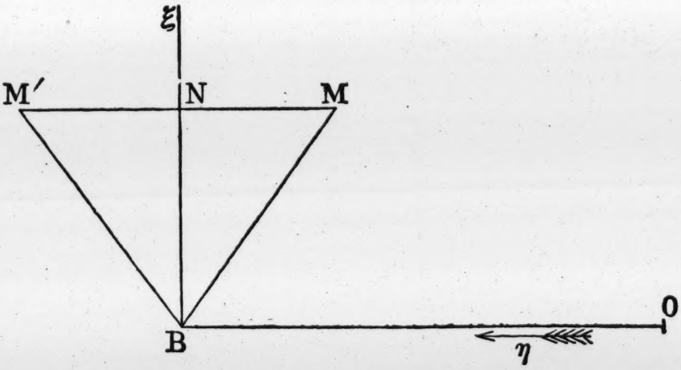
where we put

$$d\tau = 2\eta S\eta(\rho - \rho_e).$$

The displacement $d\rho$ is thus represented by two components at right angles to each other: $d\rho_e$ being constant for any point ρ, and $d\tau$ representing the double of the distance of the extremity of ρ from the plane (38) in which the extremity of ρ_e is situated.

As ρ_e is arbitrary in this plane, we may take ηC in its place, so that

$$d\tau = 2\eta S\eta(\rho - \eta C) = 2\eta(S\eta\rho + C).$$



If BM be the projection of $\rho - \eta C$ on a plane passing through O and containing the direction η, and if Bξ represents in the plane of projection the intersection with the plane (38), then $MN = \eta S\eta(\rho - \eta C)$ and $d\tau = 2\overline{MN} = \overline{MM'}$.

The points M and M′ at both extremities of $d\tau$ are therefore symmetrical in respect to the plane (38), and we will call therefore that plane the *plane of symmetry*. The component $d\tau$ having displaced the point ρ into its position of symmetry $\rho + d\tau$, the component $d\rho_e$ parallel to that plane (because perpendicular to η), and being applied to every point, will shift the system of points $\rho + d\tau$ out of their position of symmetry in causing them to move by a translation parallel to that plane, definite in direction and distance. We may designate the result of both the displacements $d\tau$ and $d\rho_e$ by the name of *perversion*.

For the second solution we have also the expressions (3) which we may write now for p and q co-axial (by 11) and apply to

$$\alpha = \rho_1 - \rho_c, \quad \beta = \rho - \rho_c,$$
$$\alpha' = \rho'_1 - \rho'_c, \quad \beta' = \rho' - \rho'_c.$$

We will now determine the angles u and v of respectively p and q. Having

$$(39) \qquad \begin{cases} \alpha' = p\alpha p^{-1}, \\ \beta' = q\beta q^{-1}, \end{cases}$$

we get, by (5) and (20),

$$(40) \qquad \begin{cases} d\alpha = 2 \sin u \,.\, p\mathrm{V}\zeta\alpha = 2\eta\mathrm{S}\eta\alpha, \\ d\beta = 2 \sin v \,.\, q\mathrm{V}\zeta\beta = 2\eta\mathrm{S}\eta\beta. \end{cases}$$

Treating these relations by S . ζ we get, as the second members vanish,

$$(41) \qquad \begin{cases} \mathrm{S}\zeta d\alpha = 0, \\ \mathrm{S}\zeta d\beta = 0, \end{cases}$$

and consequently also

$$\mathrm{S}\zeta\eta = 0,$$

Thus ζ must be in a plane perpendicular to η. We call ξ the unit-rector perpendicular to ζ and to η, so that

$$\eta\zeta = \xi,$$

and the system ξ, η, ζ will be trirectangular, ξ being also in the same plane with ζ, and to the left of it when seen from a point of the positive direction of η. Let us take the origin of that system at the extremity of ρ_c in the plane of symmetry (38).

In treating (40) by V.ζ, and remarking that $\zeta p = p\zeta$, and that $\mathrm{V}\zeta p\mathrm{V}\zeta\alpha = p\mathrm{V}\zeta\mathrm{V}.\,\zeta\alpha$, we get (by a change of sign, in replacing V.$\zeta\alpha$ by V.$\,\alpha\zeta$)

$$2 \sin u \,.\, p\mathrm{V}(\zeta\mathrm{V}\alpha\zeta) = 2\xi\mathrm{S}\eta\alpha,$$
$$2 \sin v \,.\, q\mathrm{V}(\zeta\mathrm{V}\beta\zeta) = 2\xi\mathrm{S}\eta\beta.$$

We put
$$V\zeta V\alpha\zeta = \bar{\alpha}$$
$$V\zeta V\beta\zeta = \bar{\beta},$$

remarking that $\bar{\alpha}$ and $\bar{\beta}$ represent the projections of α and β respectively on a plane perpendicular to ζ, namely, on the plans $\xi\eta$, we get, by comparison

$$\frac{2\sin u}{2S\eta\alpha} \cdot p\bar{\alpha} = \frac{\sin v}{S\eta\beta} q\bar{\beta} = \xi.$$

This gives separately

$$(42) \qquad \begin{cases} pU\bar{\alpha} = \xi\left(\dfrac{S\eta\alpha}{\sin u}\right), \\ qU\bar{\beta} = \xi\left(\dfrac{S\eta\beta}{\sin v}\right). \end{cases}$$

And we must attribute to the second members the signs respectively of $S\eta\alpha$ and of $S\eta\beta$, to be determined in each particular case. In all cases we have

$$\begin{cases} \sin u = T.\, S\eta\alpha \\ \sin v = TS\eta\beta. \end{cases}$$

So that the signs of $S\eta\alpha$ and of $S\eta\beta$ give the signs of the second members of (42).

The number of arbitrary elements, by which the displacement $d\rho$ of any point ρ may depend, will also be six in the most general solution. Having, namely,

$$(43) \qquad d\rho = d\rho_c + 2\eta S\eta(\rho - \rho_c),$$

and

$$(44) \qquad d\rho = d\rho_c + 2\sin u \,.\, PV\zeta(\rho - \rho_c),$$

we may consider ρ_c as given by ηC which contains *three* elements, two included in η, and one in C. Then η being given we have ζ by *one* more arbitary element, because ζ already has to satisfy two scalar equations,

$$S\eta\zeta = 0, \text{ and } \zeta^2 = -1.$$

When η and ζ are determined, then ξ will be known by

$$\xi = \eta\zeta.$$

Finally, the value of $d\rho_c$ is of the form

$$d\rho_c = \xi a + \zeta c,$$

where a and c constitute *two* more arbitrary elements: in all, six arbitraries, all of them entering in both the expressions (43) and (44).

It is true that in the case of one perversion one might define ζ so as to render $d\rho_c$ parallel to it, and so we would have $a=0$, and there would be only *five* arbitraries; but if we want to combine two given perversions, we are not free to dispose of the direction of ζ arbitrarily.

§ 5.

The results of the two preceding paragraphs will now be applied to the demonstration of the proposition according to which two perversions applied in succession to a given system of points produce a displacement represented by a screw-rotation.

This proposition may be looked upon as known already, but its demonstration in its greatest generality by the quaternion method has perhaps not yet been given.

We assume that the planes of symmetry corresponding to the two perversions have been determined and brought under the form

$$S\eta\tau + C = 0, \quad S\eta'\tau' + C' = 0.$$

We assume also that ρ_c, ρ'_c, ρ''_c be taken so as to assign to ρ'_c a position *on* the line of intersection of the two planes. Let moreover ρ_1, ρ, represent the vectors of two points given arbitrarily before the displacement, and let $\rho'_1\ \rho^1$ be the vectors of these points after the first perversion and ρ''_1, ρ'' the vectors of the same points after the second perversion. Then putting

$$\alpha = \rho_1 - \rho_c, \alpha' = \rho'_1 - \rho'_c, \quad \alpha'' = \rho''_1 - \rho''_c,$$
$$\beta = \rho - \rho_c, \beta' = \rho' - \rho'_c, \quad \beta'' = \rho'' - \rho_c'',$$

we may represent the perversions by

$$\alpha' = p\alpha p^{-1}, \quad \alpha'' = p'\alpha' p'^{-1},$$
$$\beta' = q\beta q^{-1}, \quad \beta'' = q'\beta' q'^{-}.$$

Hence if we put

$$p'p = p_1, \quad q'q = q_1,$$

we have

$$\alpha'' = p_1\alpha p_1^{-1},$$
$$\beta'' = q_1\beta q_1^{-1}.$$

Looking upon $\alpha'' - \alpha = d_1\alpha$, $\beta'' - \beta = d_1\beta$ as representing any displacements, we apply the conditions I. and II. of § 1 in order to determine the nature of these displacements. Generally then they must satisfy the equation

$$(46) \qquad \mathrm{T}(\mathrm{V}q_1^{-1}p_1)\mathrm{TV}(d_1\alpha d_1\beta) = 0\,.$$

Let us examine the second factor. We have as to α,

$$d_1\alpha = \alpha'' - \alpha = (\alpha'' - \alpha') + (\alpha' - \alpha) = d\alpha' + d\alpha\,.$$

Thus

$$\mathrm{V}(d_1\alpha d_1\beta) = \mathrm{V}(d\alpha + d\alpha')(d\beta + d\beta')\,.$$

But separately $d\alpha$ with $d\beta$ on the one hand, and $d\alpha'$ with $d\beta'$ on the other, give *perversions* by hypothesis. We have therefore

$$\mathrm{T.V}d\alpha d\beta = 0, \quad \mathrm{T.V}d\alpha' d\beta' = 0.$$

Thus we get

$$\mathrm{V}(d_1\alpha . d_1\beta) = \mathrm{V}.d\alpha'.d\beta + \mathrm{V}.d\alpha d\beta'.$$

Now let us consider the displacements $d\alpha$, $d\beta$, &c., under their other form (19),

$$d\alpha = 2\eta\mathrm{S}\eta\alpha\,, \quad d\alpha' = 2\eta'\mathrm{S}\eta'\alpha'\,,$$
$$d\beta = 2\eta\mathrm{S}\eta\beta\,, \quad d\beta' = 2\eta'\mathrm{S}\eta'\beta'\,.$$

These give

$$\mathrm{V}d\alpha' d\beta = 4\mathrm{V}\eta'\eta\mathrm{S}\eta'\alpha'\mathrm{S}\eta\beta\,,$$
$$\mathrm{V}d\alpha d\beta' = -4\mathrm{V}\eta'\eta\mathrm{S}\eta'\beta'\mathrm{S}\eta\alpha\,.$$

Taking the sum, and remarking that $\alpha' = \alpha + d\alpha$, $\beta' = \beta + d\beta$, we get for the factor of $4\mathrm{V}\eta'\eta$,

$$\mathrm{S}\eta'[(\alpha + 2\eta\mathrm{S}\eta\alpha)\mathrm{S}\eta\beta - (\beta + 2\eta\mathrm{S}\eta\beta)\mathrm{S}\eta\alpha]\,,$$
$$= \mathrm{S}.\eta'\alpha\mathrm{S}\eta\beta - \mathrm{S}\eta'\beta\mathrm{S}\eta\alpha\,,$$
$$= \mathrm{S}.\mathrm{V}\eta\eta'\mathrm{V}\alpha\beta\,.$$

So that

$$\mathrm{V}d_1\alpha d_1\beta = 4\mathrm{V}\eta'\eta\mathrm{S}.\mathrm{V}\eta\eta'\mathrm{V}\alpha\beta\,.$$

Generally this result cannot vanish, because β which is $\rho - \rho_c$ can be any vector.

As this factor of (46) cannot vanish, we must have

$$(47) \qquad \mathrm{TV}(q_1^{-1}p_1) = 0,$$

a solution which gives a *screw-rotation*, the operators

$$(48) \qquad p_1(\quad)p_1^{-1} \text{ and } q_1(\quad)q_1^{-1}$$

becoming identical, because the ambiguity $p_1 = \pm q_1$ disappears in the expression of the operator.

Having arrived at the condition (47), by way of exclusion we will now prove by a direct method the identity of the two operators (48).

In the preceding deductions we have admitted that p and q have the same axis, situated in a plane perpendicular to η, this plane having the definite position of the plane of symmetry. Likewise, we have also admitted that the axis of p' and q' be situated in the plane of symmetry corresponding to the second perversion. As these two planes generally intersect each other, we take the direction of the line of intersection for the axis ζ of the four versors p, q, p', q'. We have then

$$\mathrm{S}\zeta\eta = 0, \quad \mathrm{S}\zeta\eta' = 0,$$

and in putting

$$\eta\eta' = -\cos w + \zeta \sin w, \tag{49}$$

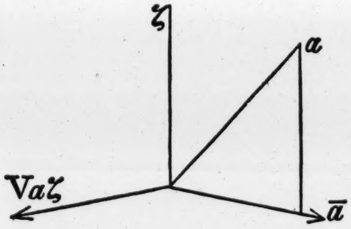
we assume that the positive half of ζ will be directed so as to have η' on the left of η, when the angle w be positive and not outpassing two right angles. Having

$$\alpha' = p\alpha p^{-1}, \quad \alpha' p = p\alpha,$$

whence we deduce $\mathrm{S}\zeta\alpha' = \mathrm{S}\zeta\alpha$, and putting

$$\bar{\alpha} = \mathrm{V}(\zeta\mathrm{V}\alpha\zeta), \quad \overline{\alpha'} = \mathrm{V}(\zeta\mathrm{V}\alpha'\zeta),$$

where $\bar{\alpha}$ and $\overline{\alpha'}$ represent the projections of α and α' on a plane perpendicular to ζ, we get



$$\overline{\alpha'}p = p\bar{\alpha}.$$

This gives $\mathrm{T}\alpha' = \mathrm{T}\bar{\bar{\alpha}}$. Hence by (42) we have

$$\overline{\alpha'}p = \xi \frac{\mathrm{S}\eta\alpha\mathrm{T}\overline{\alpha'}}{\sin u}.$$

Likewise we get, applying (42),

$$p'\overline{\alpha'} = \xi' \frac{\mathrm{S}\eta'\alpha'\mathrm{T}\overline{\alpha'}}{\sin u'}.$$

Multiplying member to member and observing $(\overline{\alpha'})^2 = -(\overline{\mathrm{T}\alpha'})^2$, we get

$$p'p = -\xi'\xi.\frac{\mathrm{S}\eta'\alpha'\mathrm{S}\eta\alpha}{\sin u'\sin u}.$$

But the same calcul, proceeding from $\beta' q = q\beta$, will establish

$$q'q = -\xi'\xi\frac{\mathrm{S}\eta'\beta'\mathrm{S}\eta\beta}{\sin v'\sin v}.$$

Both results give the same operator

$$\xi'\xi(\qquad)\xi\xi',$$

whatever the signs of the scalar factors may be in the particular expressions of $p'p$ and $q'q$.

Having $\xi = \eta\zeta = -\zeta\eta$, and $\xi' = \eta'\zeta$, we get

$$\xi'\xi = \eta'\eta.$$

We have already defined $\eta\eta'$ by (49). From that definition we deduce

$$(50) \qquad \eta'\eta^{-1} = \cos w + \zeta \sin w = r,$$

so that the operator will have the canonical form. We have thus:

$$(51) \qquad \rho'' - \rho''_c = r(\rho - \rho_c)r^{-1}$$

for all possible values of ρ.

The angle w is evidently by (50) the angle comprised by the two planes of symmetry, the angle of rotation being $= 2w$.

Let us determine the position of the axis of rotation. By the expression (26), in which we change

$$\rho_1 \text{ into } \rho_c,\ d\rho_1 \text{ into } \rho''_c - \rho_c =$$

$$= (\xi a + \zeta c) + (\xi a' + \zeta c'),$$

and u into w, p into $r = \eta'\eta^{-1}$, we get:

$$\rho_o - \rho_c = \zeta z + \frac{r^{-1}}{2\sin w}\mathrm{V}\zeta(\rho''_c - \rho_c).$$

Now we have

$$\mathrm{V}\zeta(\rho''_c - \rho_c) = \eta a + \eta' a'.$$

Hence

$$\rho_o - \rho_c = \zeta z + \tfrac{1}{2}\left[\frac{\cos w}{\sin w}(\eta a + \eta' a') + (\xi a + \xi' a')\right].$$

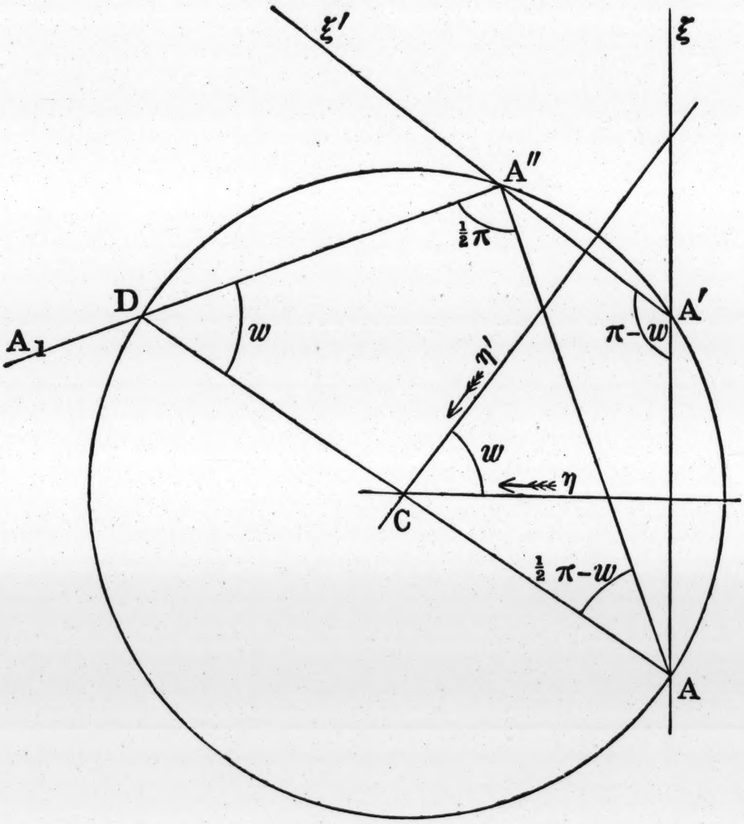
If we represent ξa by $\overline{AA'}$, $\xi' a'$ by $\overline{A'A''}$, then

$$\xi a+\xi' a'=\overline{AA''};$$

also $$\eta a+\eta' a'=\zeta\overline{AA''}=\overline{A''A_1},$$

namely at right angles to $\overline{AA''}$, and of equal length. To construct $\frac{\cos w}{\sin w}\overline{A''A_1}$ we describe the circle passing through the three points A, A′, and A″, and the point D where $A''A_1$ meets the circumference will give

$$\overline{A''D}=\frac{\cos w}{\sin w}\overline{A''A_1}.$$



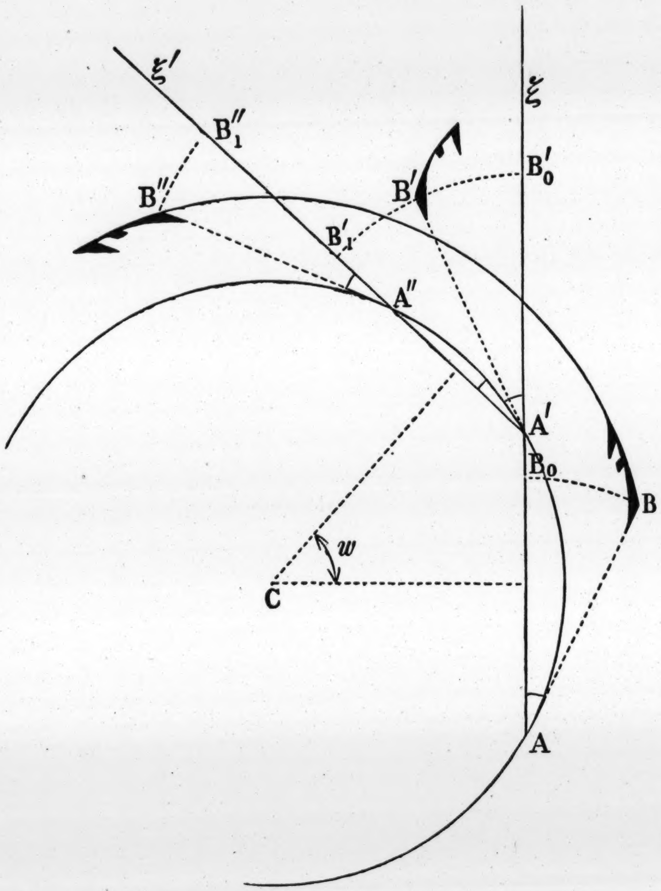
This is because the angle ADA″ is supplementary to the angle AA′A″, which itself is supplementary to w; and as the triangle AA″D is rectangular in A″ by construction, we have, for the length $AA_1=AA''$,

$$A''D=\cot w.\ \overline{AA_1}=\frac{\cos w}{\sin w}(\eta a+\eta' a'),$$

therefore

$$\rho_0 - \rho_e = \zeta z + \tfrac{1}{2}(\overline{AA''} + \overline{A''D})$$
$$= \zeta z + \tfrac{1}{2}\overline{AD} = \zeta z + \overline{AC}\,,$$

C being the centre of the circle, because AD is a diameter. In fact the construction reduces itself practically to the determination of the centre C of the circumference passing through A, A′A″; the axis of rotation being then a line passing through C and being perpendicular to the plane which is parallel to η and η' the normals of the planes of symmetry.



BUSINESS.

The following Candidates were balloted for and declared duly elected Fellows of the Society:—Dr P. M'Bryde, F.R.C.P. Ed.; Mr G. W. W. Barclay; and Mr Thomas Andrews, F.C.S.

Monday, 16th April 1883.

Mr JOHN MURRAY in the Chair.

The following Communications were read:—

1. On some Properties of the Line of Simple Flexure.

By Edward Sang, C.E. (Plates I.*–III.)

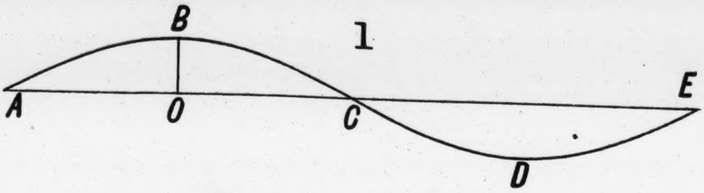
The unrestricted problem, "To find the form assumed by an elastic system when subjected to known strains," is one of those mechanical problems which baffle the powers even of the modern calculus. It is only when the change of form is exceedingly small that we can obtain approximate results. In the simplest case—that of a straight uniform elastic body—there is, so far as I know, no complete solution; and thus I venture to suppose that the following remarks may not be devoid of interest:—

If a thin flat rectangular plate—a physical line, as it were—be bent by means of a string attached to its two ends, it takes a particular form, to which the name "curve of simple flexure" may be given.

From the nature of the case, it is clear that if ABC represent the bent plate and AC the string, the curve must be symmetric from the two ends; so that if B be the middle of the bow and BO the ordinate therefrom, the two parts AOB, COB must be alike. Farther, if a second spring of the same dimensions, and bent by a string of the same length, were placed endways to this one, as at CDE, the strings AC, CE being in one straight line, but the bend being on the other side thereof, the ends may be conceived as united at C, so as to form a continuous elastic plate, while the cord may be supposed attached to the two ends A and E.

The same kind of extension may be continued indefinitely both ways; and thus it follows that the curve of simple flexure is com-

1



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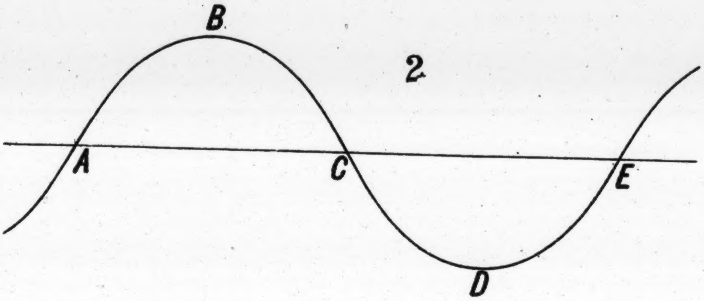

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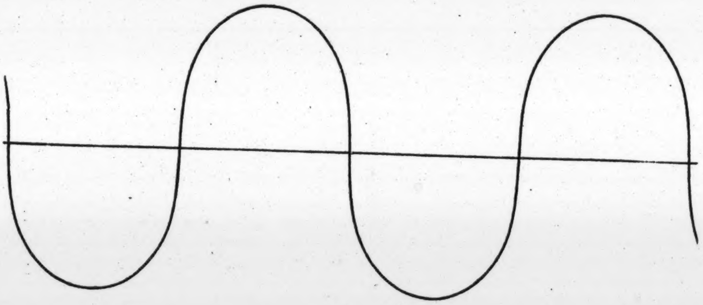

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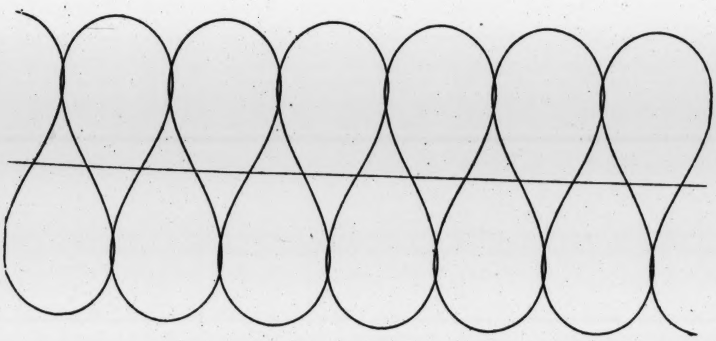

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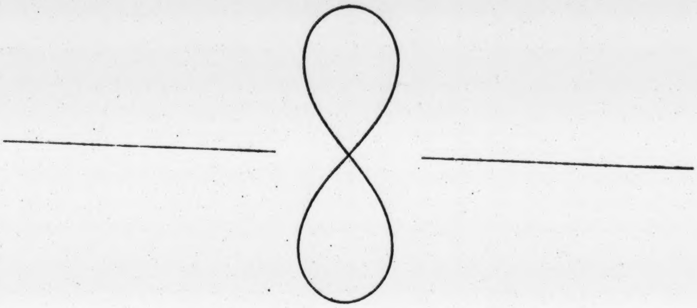

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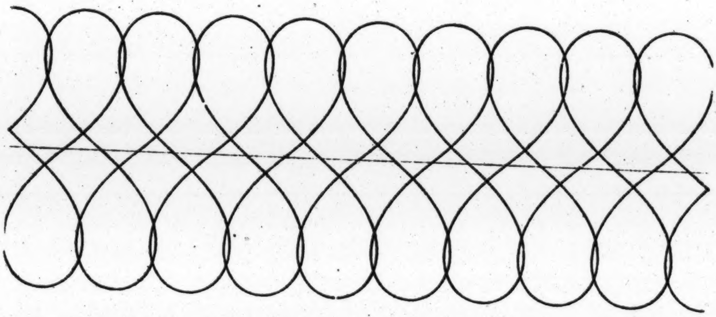

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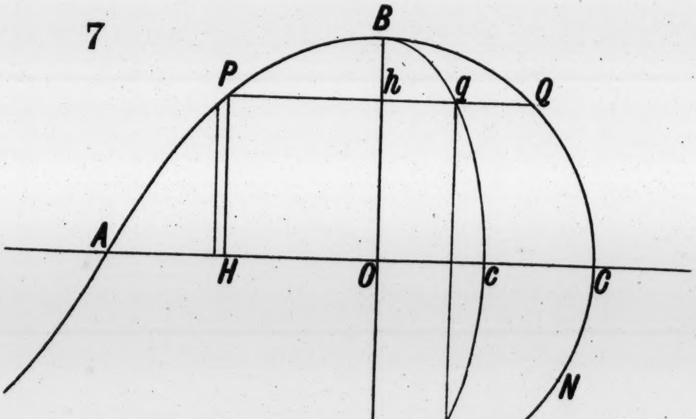
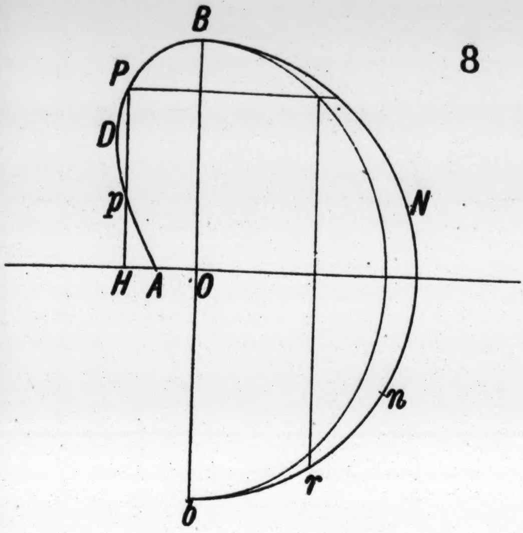


Plate. II.

8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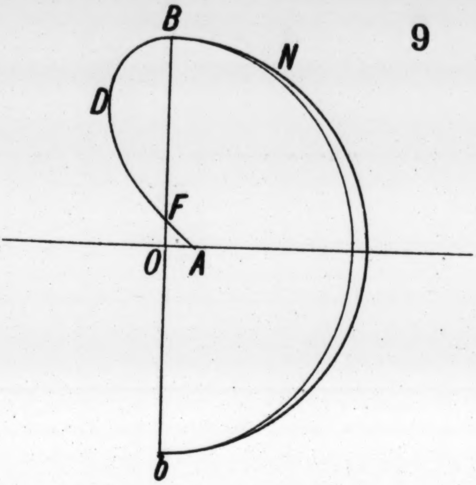
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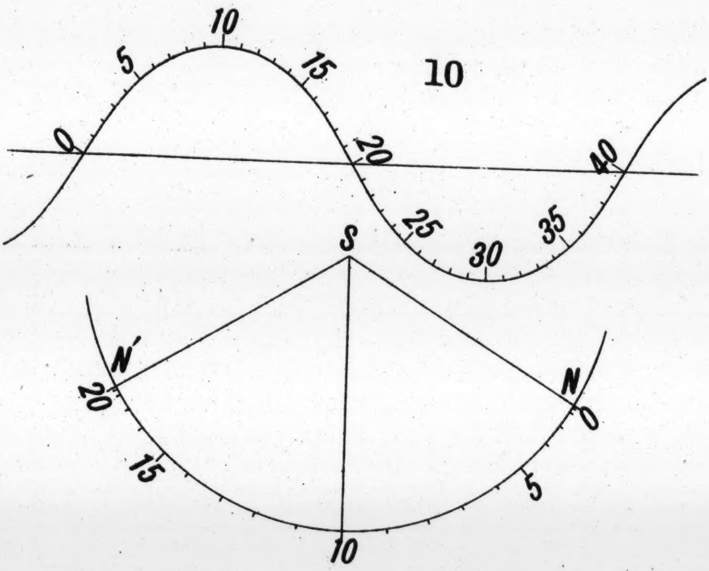
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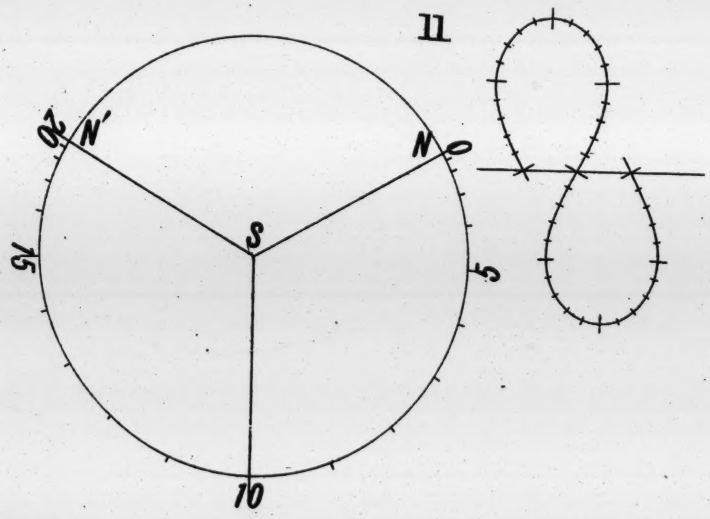
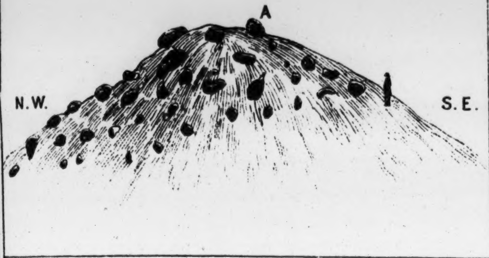

N′
N
S

Diag. 1.

TOP OF HILL ON LOCH KILLESPORT COVERED WITH BOULDERS



Diag. 2.

INDIVIDUAL BOULDERS FROM HILL SHOWN IN FIG. I
LOCH KILLESPORT

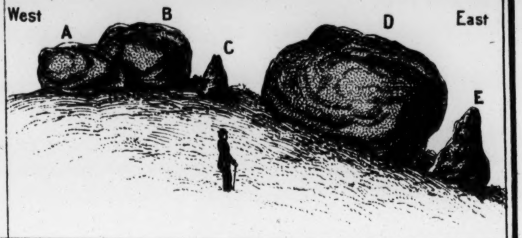
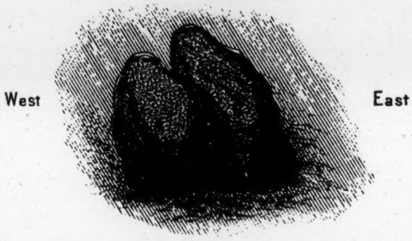


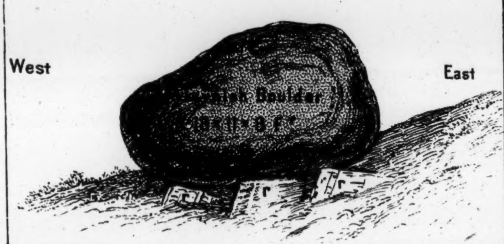
Fig. 3.

ONE BOULDER LEANING ON ANOTHER ON THE HILL IN FIG. I
LOCH KILLESPORT

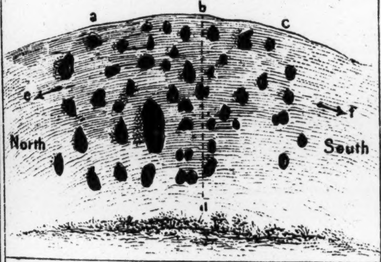


Diag. 4.

LOCH KILLESPORT



TAYNISH LOCH KILLESPORT Diag. 5.
SMOOTHED ROCK COVERED BY BOULDERS
ROCK FACE SLOPES DOWN WESTWARD



LOCH SWEYN Fig. 7.
ROCK SURFACE SMOOTHED AND STRIATED,
WITH HOLES ON SURFACE
Length about 25 Ft. Breadth 8 Ft.

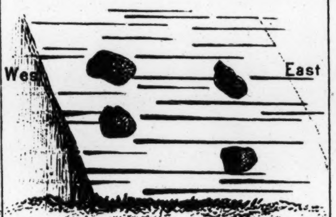
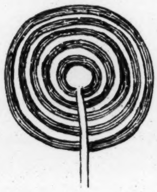
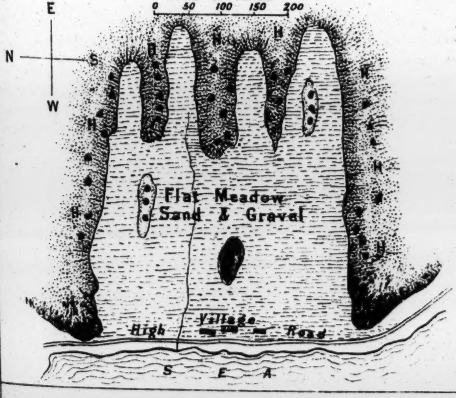


Fig. 9

SPECIMEN OF CUP MARKS
ON GLACIATED ROCKS
LOCH GILP HEAD 1 FT WIDE



LOCH KILLESPORT Diag. 6.



LOCH SWEYN Diag. 8.

BOULDERS ON HILL SIDE, HEAPED ON ONE ANOTHER

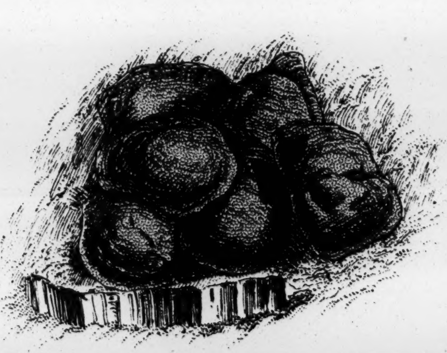
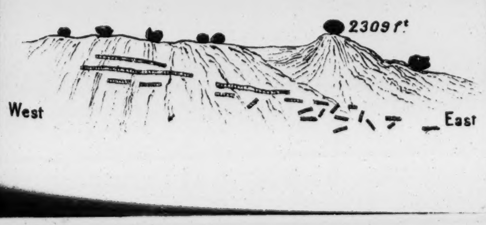
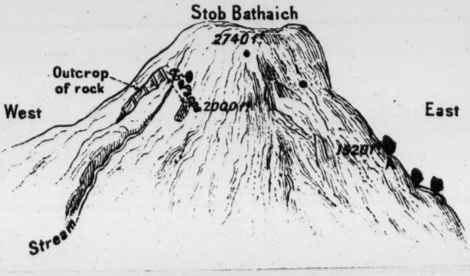


Fig. 10

BOULDERS, PERCHED ON RIDGE, & SUMMIT OF A HILL
BEIN-NAN-CLACH, GLEN DOCHART, PERTHSHIRE



LOCH CLUNIE DISTRICT, INVERNESS-SHIRE Fig. 11



posed of an endless succession of equal waves, disposed alternately on either side of a straight line. It belongs, then, to the class of transcendental curves typified by the curve of sines.

Instead of the string we may put two obstacles, one at each end, against which the spring may press. It is obvious that the arrangement ABCDE would be one of unstable equilibrium, so that some slight guide would need to be placed at C, in order to prevent the spring from flying to the one or to the other side.

When the bending is slight, as shown in the first figure (Plate I.*), the form bears a considerable resemblance to that of the curve of sines when flattened to the same degree; but when the flexure is considerable, as in the second figure, the deviation from that form becomes marked; the angle of crossing at the points A, C, E, necessarily is increased. The third figure shows the form of the spring, with its continuation, when the ends are so drawn together, as that the curve crosses the axis squarely.

In the fourth figure the spring is shown as so much bent that the angle of crossing is obtuse; the distance between the ends is less than the breadth of the loop at its widest part. In the actual figure this distance is less than half of the breadth, and hence the continuations of the loops intersect each other.

When the ends are brought still nearer the loops come to cross each other more frequently.

The fifth figure shows the form of the spring when the two ends are brought together. In this case the continuations of the form are all included in its counterpart on the other side and in itself.

When the ends of the spring are crossed over each other, it takes the form of what is technically called a *kink*, as shown in the sixth figure. There the distance of the ends is less than half the width of the loop, and the continuations intersect each other. If that distance were augmented the loops would stand detached.

In these changes we have a noteworthy instance of the danger of abstract reasoning as to limits. While the point C is being brought nearer to A, the number of undulations of the curve within the limits of the sheet increases, so much so, that if C were very close to A, the paper would be covered by a multitude of lines, which, however fine they may be, would tend to produce blackness.

If, after the spring has been crossed, we allow C to come back to

A, we shall have a corresponding increase in the number of undulations comprised within the limits of the sheet; the tendency again being toward blackness. On making the approach from either side, that is, on taking the functions of $a+\delta a$ and of $a-\delta a$, and attributing to the variations δa an infinitesimally small value, we find, on both sides, the functions to be black; yet, on making δa absolutely zero, we find whiteness instead of blackness.

Such being the general features of the curve, we may obtain its details by an examination of one of the half-waves, such as AOB. For this analysis it will be most convenient to place the origin of co-ordinates at the point O. We shall therefore write—

$y=$ OH
$x=$ HP
$l=$ arc BP
$r=$ radius of curvature at P
$a=$ inclination of curve at P
$s=$ surface BOHP.

We shall also write—

$Y=$ OA
$X=$ OB
$L=$ length of BA
$R=$ radius of curvature at B
$A=$ inclination at A
$S=$ area BAO,

for the limits of these quantities.

Since, in such an arrangement, the angular tension at the point P is proportional to the ordinate HP, the radius of curvature there must be inversely proportional to the same ordinate; and therefore we must have

$$rx=c^2 \quad . \quad . \quad . \quad . \quad (1)$$

where c is a constant determined by the dimensions of the spring. This equation contains the analytical definition of the curve.

Since, from the nature of curvature,

$$dl=r\,.\,da$$

the generic equation (1) may be written

$$x\,.\,dl=c^2.\,da\,, \quad . \quad . \quad (2)$$

but

$$dy = \cos a \,.\, dl$$

wherefore

$$x \,.\, dy = c^2 \,.\, \cos a \,.\, da\,;$$

but the product $x \,.\, dy$ represents the increment of the area BOHP, wherefore we have

$$S = c^2 \,.\, \sin a \quad . \quad . \quad . \qquad (3)$$

whence also

$$S = c^2 \,.\, \sin A\,.$$

When the angle A is acute, as in fig. 7, there is no difficulty in interpreting this equation; but when that angle is obtuse, as in fig. 8, we have to observe that, in supposing the point P to move along the curve from B, carrying with it the ordinate PH, the area OBPH increases until P arrive at D, where the curve becomes perpendicular to the axis. Beyond that limit the motion is towards BO, the differential of the area becomes subtractive, and thus, for the point p in the figure, s would represent the surface BPDpHO, while S would stand for the area BPDpAO. And when the spring is crossed so as to take A to the other side of O, as in fig. 9, the surface AFO must be regarded as subtractive, so that the symbol S then stands for the excess of BDF above FOA.

On inserting the value of dl, viz.,

$$-dl = dx \,.\, \sin a^{-1}$$

into the equation (2), we obtain

$$-xdx = c^2 \,.\, \sin a \,.\, da$$

which, when integrated, becomes

$$\tfrac{1}{2}x^2 = +c^2 \,.\, \cos a + \text{constant}\,.$$

Now, when $x = 0$, a becomes A, wherefore the value of the constant is $-c^2 \cos A$; so that

$$\tfrac{1}{2}x^2 = c^2(\cos a - \cos A)\,, \quad . \quad . \qquad (4)$$

which equation completely determines the relation of a the inclination, to x the ordinate of the curve.

Here we observe that when P is at B, a becomes zero and x becomes X; wherefore

$$\tfrac{1}{2}X^2 = c^2(1 - \cos A)\,, \text{ which gives}$$

$$c^2 = \frac{X^2}{2(1 - \cos A)} = \frac{X^2}{4(\sin \frac{1}{2}A)^2} \quad . \quad . \qquad (5)$$

and consequently

$$\left.\begin{aligned}\frac{x^2}{X^2}&=\frac{(\sin\frac{1}{2}A)^2-(\sin\frac{1}{2}a)^2}{(\sin\frac{1}{2}A)^2}\\ \text{or}\qquad \frac{X^2-x^2}{X^2}&=\left(\frac{\sin\frac{1}{2}a}{\sin\frac{1}{2}A}\right)^2\end{aligned}\right\}\quad .\quad .\qquad (6)$$

If, from O as a centre, we describe the semicircle BC*b* (fig. 7), and draw P*h*Q parallel to the axis, we have $hQ^2=OB^2-HP^2=X^2-x^2$, wherefore OC : *h*Q : : sin $\frac{1}{2}$A : sin $\frac{1}{2}a$; and hence if, having made O*c* equal to sin $\frac{1}{2}$A to the radius OB, we describe the semi-ellipse B*cb*, the portion *hq* intercepted by it must be equal to sin $\frac{1}{2}a$ to the same radius. Hence, if we draw Q*r* parallel to B*b*, the angle *b*O*r* must be $\frac{1}{2}a$; and if we make *rn* equal to *br*, the radius O*n* must be perpendicular to the straight line touching the curve at P, or, in other words, the direction of the circumference at *n* is parallel to that of the curve at P.

Similarly, if a parallel to B*b* were drawn through *c*, and the arc intercepted from *b* doubled, we should get the limiting position N, where the circumference is parallel to the curve at A, that is to say, the angle *b*ON is equal to the maximum inclination of the curve.

Thus it is easy to obtain, by calculation or by construction, the value of x corresponding to a given inclination, or the inclination corresponding to a given value of x, when OB and the maximum inclination A are known.

We may thus obtain an approximation to the true form of the curve by a graphic process. A number of lines parallel to the axis, and crossing OB, having been drawn, we ascertain the angle at which each of them should be crossed by the curve, and draw a series of short connected lines with their inclinations. In this operation we have our choice among three processes. We may divide the ordinate OB equally, computing the corresponding inclinations; we may divide the inclinations equally, computing the successive values of x; or else we may graduate the quadrant BQC equally. The last-named arrangement is the most convenient of the three.

A still closer approximation may be made by considering the curvature. From equation (5) we at once get the value of c^2, that is, of the product rx, so that the radius of curvature is easily computed; and we are able to compose a series of short circular arcs to represent the curve very closely. It may be remarked that when A is 60°, the circle osculating the curve at B has O for its centre; and that when the crossing is at right angles, the radius of curvature at the vertex is just one-half of the major ordinate.

Since the whole area ABO is expressed by $c^2 \sin A$, and OBPH by $c^2 \sin a$, we have

$$\mathrm{APH} = c^2(\sin A - \sin a)$$

but

$$\tfrac{1}{2}\mathrm{PH}^2 = c^2(\cos a - \cos A)$$

wherefore

$$2\mathrm{APH} : \mathrm{OB}^2 :: \sin A - \sin a : \cos a - \cos A :: 1 : \tan\frac{A+a}{2} \,,$$

so that

$$2 \,.\, \mathrm{ABO} : \mathrm{OB}^2 :: 1 : \tan\frac{A}{2} \,.$$

And thus, when the crossing is perpendicularly, as in fig. 3, the area of one wave of the curve is just equivalent to the square of the major ordinate.

Equation (4) gives us

$$x = c\sqrt{2}\,\sqrt{(\cos a - \cos A)} \,,$$

so that the second equation becomes

$$dl = \frac{c}{\sqrt{2}}(\cos a - \cos A)^{-\frac{1}{2}}da \,,$$

and thus the length BP of the curve is to be got by the integration of this transcendental expression, that is,

$$l = \frac{c}{\sqrt{2}}\int(\cos a - \cos A)^{-\frac{1}{2}}da \,,$$

or

$$l = \frac{\mathrm{X}}{cho\mathrm{A} \,.\, \sqrt{2}}\int(\cos a - \cos A)^{-\frac{1}{2}}da \,.$$

If we suppose the plane of the paper to be placed vertically, OB being directed toward the zenith, and if we imagine ON to represent the extreme position of a simple pendulum, the velocity acquired in descending from N to n (Pl. II. fig. 7) would be proportional to $\sqrt{(\cos a - \cos A)}$, and thus the element of time in which an increment da of the arc is passed over would be $(\cos a - \cos A)^{-\frac{1}{2}}da$, and we thus have this remarkably beautiful theorem.

If, on the same plane with the line of simple flexure ABCDE, a circle be described round S, the point of suspension of a supposed pendulum, and if the points, P in the curve and n in the circle, be so related that the directions of the two curves be always parallel to each other, then a uniform motion of P along the curve ABCDE (Pl. III. figs. 10, 11) will be accompanied by an oscillatory motion of n (from N to N′ and back), exactly in imitation of the motion of a pendulum.

In the twenty-fourth and twenty-sixth volumes of the Society's *Transactions*, I have described a very expeditious process for computing the motion of a heavy body along the circumference of a circle, which process, by an obvious modification, enables us to get the integral

$$\int(\cos a - \cos \mathrm{A})^{-\frac{1}{2}} da$$

in any proposed case. Thus we may obtain the length of the curve intercepted from the vertex B to any proposed horizontal line.

It remains for us to investigate the relation of the abscissa y to the inclination. Substituting for x its value, we obtain from the above equation

$$xdy = c^2 . \cos a . da ,$$

$$dy = \frac{c}{\sqrt{2}} \cos a(\cos a - \cos \mathrm{A})^{-\frac{1}{2}} da$$

whence

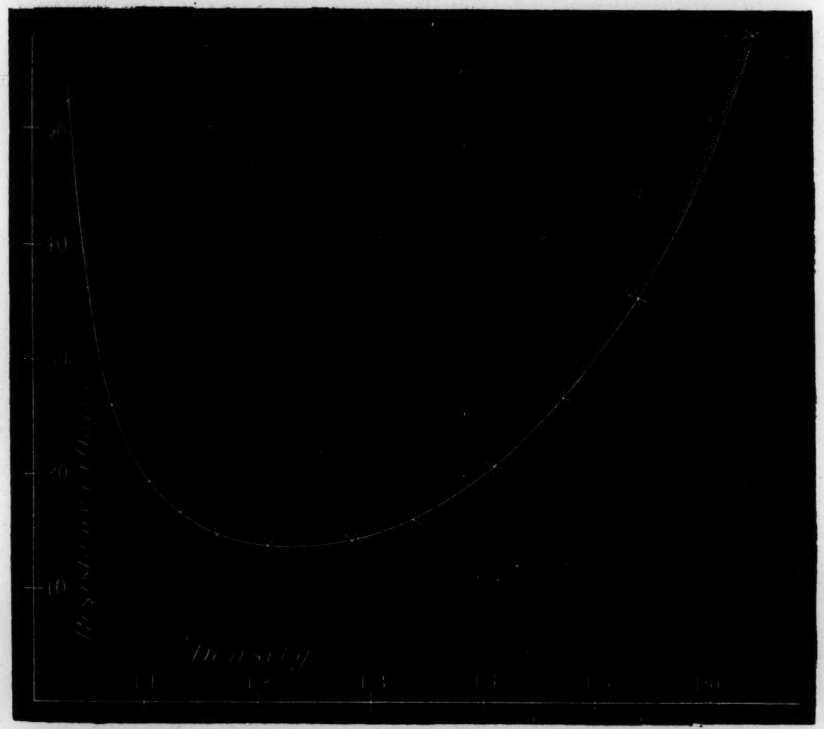
$$y = \frac{c}{\sqrt{2}} \int (\cos a - \cos \mathrm{A})^{-\frac{1}{2}} \cos a . da ,$$

and thus the problem to determine the form of the curve of flexure by help of its co-ordinates resolves itself into a transcendental integration.

2. On the Measurement of Resistance to Electrolytes. By Cargill G. Knott, D.Sc., F.R.S.E.

The difficulties attending the measurement of the electrical resistance of electrolytes are well known, the rapid growth of the polarisation of the electrodes especially preventing the application of the ordinary Wheatstone Bridge method. The polarisation may be kept down by using alternating currents, as was done by Kohlrausch; and this method is no doubt the most general and most accurate which has yet been applied.

It occurred to me some years ago that electrometer measurements might give results sufficiently accurate for most practical purposes. To measure the difference of potential between the electrodes by which the current enters and leaves the electrolyte is of course out of the question; but by dipping into the liquid two otherwise insulated platinum points which are connected to the electrometer, we may minimise the effect of polarisation.



The apparatus used in the experiments to be described was constructed fully a year ago, and was simply a horizontal glass tube (11·8 cm. long) fused into the sides of two vertical test-tubes, and having two platinum wires fused into it at points distant 7·3 cm. from each other. The platinum points just projected into the inside of the tube, the bore of which was 0·9 sq. cm. in cross-section. The electrolyte stood in the vertical tubes at a sufficient height to fill the horizontal connecting tube completely; and into the vertical tubes platinum plates were inserted to act as the current electrodes. A current from two Bunsen cells was driven through the electrolyte and a standard coil of 10 ohms. A rocking commutator permitted the electrodes of the Thomson quadrant electrometer to be connected either to the ends of the standard coil, or to the platinum wires that came from the electrolyte. In this way, a simple comparison of deflections while a steady current was flowing gave the ratio of the resistances of the standard coil and the liquid column between the platinum points.

When the current was kept steadily in one direction, the platinum points became gradually polarised to different potentials; but yet the *difference* between the reading when the current was flowing, and the reading when the current was stopped, was very constant, whatever this latter reading (the approximate zero) might be. By reversing the current from time to time, this polarisation at the points could be kept down; and in no case was the approximate zero ever so great as to show any tendency to fall off appreciably when left for several minutes to itself.

A few preliminary experiments convinced me of the possibility of obtaining results by this method; but it is only within the last few months that a complete series of experiments have been carried out, for which I have to thank Messrs J. W. Macdonald and G. Gregory Smith, students in the Physics Library of the University. These gentlemen investigated very carefully the variation of resistance with density of solutions of sulphuric acid and water at the ordinary temperature (about 10° C.). The following table gives the final reductions:—

Resistance.	Density.
57·87 ohms.	1·647
35	1·540
26·74	1·470
20·57	1·410
15·55	1·340
14·18	1·282
13·9	1·204
14·44	1·164
16·47	1·130
19·36	1·098
25·93	1·069
52·19	1·033

The graphical representation of these numbers gives a good curve, giving 1·23 for the density of the solution of least resistance.

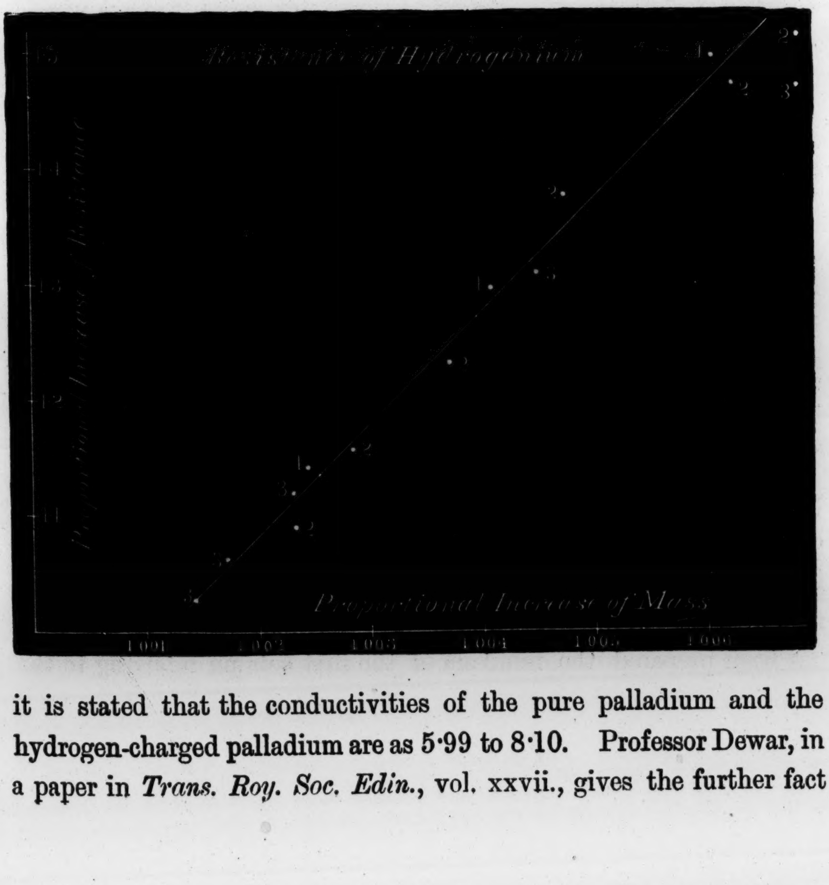
The specific resistances given by these experiments are all considerably too high, and the more so the greater the difference of potential between the points. I think it highly probable, however, that with a longer liquid column, and a stronger current, more accurate values would be obtained.

3. The Electrical Resistance of Hydrogenised Palladium.

By Cargill G. Knott, D.Sc., F.R.S.E.

That the electrical resistance of palladium is increased by hydrogenisation has long been known, but the lack of any very definite information induced me to investigate the matter last winter.

In one of Graham's papers in Poggendorff's *Annalen* for 1869, it is stated that the conductivities of the pure palladium and the hydrogen-charged palladium are as 5·99 to 8·10. Professor Dewar, in a paper in *Trans. Roy. Soc. Edin.*, vol. xxvii., gives the further fact



that the increase of resistance is proportional to the charge, without, however, detailing any particulars.

In the experiments now to be described, the palladium wire was charged by being made the negative electrode of an electrolytic cell. The wire was removed from time to time, and its resistance and mass both measured. The palladium was purified of the hydrogen by being heated to a white heat in a Bunsen flame—a method which, though very effective, was rather destructive to the palladium, which, after several chargings and dischargings, became rent and fissured in an extraordinary manner.

The following are the results of three successive experiments made with the same wire before it became so disfigured as to be practically useless for the purpose. The mass is in grammes, the resistance in ohms:—

EXPERIMENT I.—November 27, 1882.

Mass, ...	3·562	3·5702	3·5766	3·5835	3·586
Resistance, . . .	·193	·2215	·2515	·288	·289

EXPERIMENT II.—November 28, 1882

Mass, . .	3·559	3·567	3·569	3·572	3·576	3·581	3·583	3·583
Resistance, .	·1865	·204	·218	·232	·2575	·275	·283	·283

EXPERIMENT III.—November 29, 1882.

Mass, . . .	3·557	3·562	3·563	3·565	3·573	3·579
Resistance, . .	·1835	·1885	·195	·206	·242	·270

The first member for each row refers to the pure uncharged palladium. Hence, dividing every number by the first in the corresponding row, numbers representing the proportional increase in the mass and resistance will be obtained, and the different experiments made directly comparable. In this way the following table has been prepared, the numbers of the first column referring to the experiment from which the other corresponding numbers have been derived. The numbers of the first row are of course common to all three experiments:—

Experiment.	Mass.	Resistance.
1, 2, 3,	1	1
3,	1·0014	1·027
3,	1·0017	1·062
2,	1·0023	1·094
3,	1·0023	1·122
1,	1·0024	1·148
2,	1·0028	1·169
2,	1·0037	1·244
1,	1·0041	1·303
3,	1·0045	1·319
2,	1·0048	1·381
1,	1·0060	1·494
2,	1·0062	1·474
1,	1·0068	1·499
2,	1·0068	1·518
2,	1·0068	1·518
3,	1·0068	1·472

Graphically represented, these points lie clustered pretty closely round a straight line inclined to the axis, along which mass is measured at an angle whose tangent is ·955, but this straight line does not pass through the origin. Hence it would appear that the resistance does not *begin* to grow so quickly, but that from the point at which the mass has become ·1 per cent. greater than at first the increase of resistance is directly proportional to the increase of mass.

The ratio of the masses of saturated and pure palladium agrees well with Dewar's results. The ratio of the conductivities is almost exactly as 2 to 3, somewhat greater than that cited above.

When the palladium so hydrogenised was used as one element of a simple cell, of which dilute sulphuric acid and platinum were the other elements, the electro-motive force, as measured on the Thomson quadrant electrometer, was found to vary very curiously in relation to the hydrogen charge. The results are given in the following table, the unit mass referring to the pure palladium as above. The second column is the difference of potential in volts between the palladium and platinum poles:—

Mass.	Electro-motive Force.
1	− ·05
1·0014	+ ·81
1·0017	+ ·79
1·0023	+ ·73
1·0045	+ ·74
1·0068	+ ·33

A slight charge of hydrogen makes the palladium strongly positive to the platinum, but this characteristic gradually diminishes till, finally, when the palladium is fully charged with hydrogen, the electro-motive force is less than half its original amount. This fact seems new, and deserves closer study.

4. Note on Plane Algebra. By A. Macfarlane, M.A., D.Sc.

By Plane Algebra I mean what De Morgan called Double Algebra. While ordinary algebra deals with quantities which are represented on a straight line, and Quaternions with quantities which are represented in space, Double Algebra deals with those which are represented on a plane. The object of this paper is to show some applications of this intermediate method.

The quantities considered are conveniently denoted by small Roman letters, leaving their Tensor component to be denoted by the corresponding Italic letter, and the Versor component by the corresponding Greek letter. Thus a denotes a line of length a and angle α; b a line of length b, and angle β. Quantities of this kind are related to those of ordinary algebra as genus and species, and the laws of operation for the former are very easily generalised from those for the latter.

Expansions can be obtained by altering the order of the operations performed; for example, first, by applying the Binomial Theorem and then resolving; and second, by resolving and then applying the Binomial Theorem.

For example—

$$\frac{1}{\mathrm{a}-\mathrm{b}}=\frac{1}{\mathrm{a}}\left(1-\frac{\mathrm{b}}{\mathrm{a}}\right)^{-1}=\frac{1}{\mathrm{a}}+\frac{\mathrm{b}}{\mathrm{a}^2}+\frac{\mathrm{b}^2}{\mathrm{a}^3}+$$

$$=\frac{1}{a}\cos\,(-\alpha)+\frac{b}{a^2}\cos\,(\beta-2\alpha)+\frac{b^2}{a^3}\cos\,(2\beta-3\alpha)+$$

$$+i\left\{\frac{1}{a}\sin\,(-\alpha)+\frac{b}{a^2}\sin\,(\beta-2\alpha)+\frac{b^2}{a^3}\sin\,(2\beta-3\alpha)+\right\}.$$

Again,

$$\frac{1}{\mathrm{a}-\mathrm{b}}=\frac{1}{a\cos\alpha-b\cos\beta+i\,(a\sin\quad-b\sin\beta)}$$

$$=\frac{1}{a\cos\alpha-b\cos\beta}\left\{1-i\,\frac{a\sin\alpha-b\sin\beta}{a\cos\alpha-b\cos\beta}\right.$$

$$\left.+\,i^2\left(\frac{a\sin\alpha-b\sin\beta}{a\cos\alpha-b\cos\beta}\right)^2-i^3\left(\frac{a\sin\alpha-b\sin\beta}{a\cos\alpha-b\cos\beta}\right)^3+\right\}$$

Hence, by equating the components along the initial axis,

$$\frac{1}{a\cos\alpha-b\cos\beta}\left\{1-\frac{a\sin\alpha-b\sin\beta}{a\cos\alpha-b\cos\beta}\right)^2+\left(\frac{a\sin\alpha-b\sin\beta}{a\cos\alpha-b\cos\beta}\right)^4-\left.\vphantom{\frac{1}{1}}\right\}$$

$$=\frac{1}{a}\cos\alpha+\frac{b}{a^2}\cos(2\alpha-\beta)+\frac{b^2}{a^3}\cos(3\alpha-2\beta)+\,.$$

Another identity is obtained by equating the components along the perpendicular axis.

By treating $(1+\mathrm{a})^{\frac{1}{2}}$ in a similar manner we get

$$1+\frac{1}{2}\,a\cos\alpha-\frac{1}{2\,.\,4}\,a^2\cos 2\alpha+\frac{1\,.\,3}{2\,.\,4\,.\,6}\,a^3\cos 3\alpha-$$

$$=(1+a\cos\alpha)^{\frac{1}{2}}\left\{1+\frac{1}{2\,.\,4}\left(\frac{a\sin\alpha}{1+\cos\alpha}\right)^2-\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6\,.\,8}\left(\frac{a\sin\alpha}{1+\cos\alpha}\right)^4+\right\}$$

and

$$\frac{1}{2}\,a\sin\alpha-\frac{1}{2\,.\,4}\,a^2\sin 2\alpha+\frac{1\,.\,3}{2\,.\,4\,.\,6}\,a^3\sin 3\alpha-$$

$$=(1+a\cos\alpha)^{\frac{1}{2}}\left\{\frac{1}{2}\,\frac{a\sin\alpha}{1+a\cos\alpha}-\frac{1\,.\,3}{2\,.\,4\,.\,6}\left(\frac{a\sin\alpha}{1+a\cos\alpha}\right)^3+\right\}.$$

An expansion for $\log\{a^2+b^2+2ab\cos\theta\}^{\frac{1}{2}}$ is derived as follows:—

$$\mathrm{Log}\,(\mathrm{a}+\mathrm{b})=\log\mathrm{a}+\log\left(1+\frac{\mathrm{b}}{\mathrm{a}}\right).$$

Now

$$\log\mathrm{a}=\log a+i\log\alpha\,,$$

and

$$\log\left(1+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{b}}{\mathrm{a}}-\frac{1}{2}\left(\frac{\mathrm{b}}{\mathrm{a}}\right)^2+\frac{1}{3}\left(\frac{\mathrm{b}}{\mathrm{a}}\right)^3-$$

$$=\frac{b}{a}\cos(\beta-\alpha)-\frac{1}{2}\left(\frac{b}{a}\right)\cos 2(\beta-\alpha)+$$

$$+i\left\{\frac{b}{a}\sin(\beta-\alpha)-\frac{1}{2}\left(\frac{b}{a}\right)\sin 2(\beta-\alpha)+\quad\right\}.$$

Also

$$\log\{\mathrm{a}+\mathrm{b}\}=\log\left[\left(a^2+b^2+2ab\cos(\beta-\alpha)\right)^{\frac{1}{2}}.\tan^{-1}\frac{a\sin\alpha+b\sin\beta}{a\cos\alpha+b\cos\beta}\right]$$

$$=\frac{1}{2}\log\left(a^2+b^2+2ab\cos(\beta-\alpha)\right)+i\log\tan^{-1}\frac{a\sin\alpha+b\sin\beta}{a\cos\alpha+b\cos\beta}$$

Equate the components along the initial axis, and put $\beta-\alpha=\theta$.

The direct logical power of the method is illustrated by the mode in which it deduces the expressions for the acceleration along and perpendicular to the radius vector for a point moving in any plane curve from the expression for the velocity;

$$\mathrm{r}=r\,.\,\theta$$

Given

$$\frac{d\mathrm{r}}{dt}=dr\,.\,\theta+ird\theta\,.\,\vartheta\,,$$

then apply that principle again,

$$\frac{d^2\mathrm{r}}{dt^2}=d^2r\,.\,\theta+idrd\theta\,.\,\theta+idrd\theta\,.\,\theta+ird^2\theta\,.\,\theta+i^2rd\theta^2\,.\,\theta$$

$$=\left(d^2r-r(d\theta)^2\right).\,\theta+i(2drd\theta+rd^2\theta)\,.\,\theta\,.$$

5. On Heat-Conduction in Heterogeneous Bodies, as modified by the Peltier and Thomson effects. By Professor Tait.

6. Note on the Thermo-electric Position of Pure Ruthenium. By Professor Tait.

Monday, 7th May 1883.

Professor DOUGLAS MACLAGAN, M.D., Vice-President, in the Chair.

At the request of the Council, Professor Geikie delivered an Address on Recent Advances in European Pleistocene Geology.

BUSINESS.

The following Candidates were balloted for and declared duly elected Fellows of the Society:—Professor Butcher; Mr G. M'Roberts, F.C.S.; Mr R. W. Felkin, F.R.G.S.; and Mr G. Leslie, M.B., C.M.

Monday, 21st May 1883.

Mr ROBERT GRAY, Vice-President, in the Chair.

The Chairman read Communications from the Science and Art Department, and from the Education Department.

The Chairman read Obituary Notices of Sheriff Frederick Hallard, Dr John Muir, Friedrich Wöhler, and Sir John Rose Cormack, deceased Fellows of the Society.

The following Communications were read:—

1. On the Moon and the Weather. By John Aitken.

When residing in the south of France lately, I happened to look at the new moon one evening through the clear air of the "Mistral," which was blowing at the time, and not being able to see the dark body of the moon, it all at once struck me that something more was necessary than a clear atmosphere in order to enable us to see the dark side of the moon, and that the dark side would be best seen when the earth was to a great extent covered with clouds.

When we look at the moon when it is a few days old we see that the part of it turned towards the sun is brilliantly illuminated, and that the part in shadow is dark. Further, the degree of illumination of the dark part is known to vary, and is generally supposed to be brightest when our atmosphere is clearest.

Now it is evident that while the sun can illuminate the side of the moon turned towards it, it is quite unable to throw any light on the parts in shadow, as there is no atmosphere or anything round the moon to reflect the light to the shadows. The result is, that so far as the direct rays of the sun are concerned, the dark side of the moon would be quite invisible to us.

If we now transport ourselves in imagination to any part of the moon's surface that is in shadow, we would see our earth like a large moon, which would wax and wane exactly as the moon appears to us, only in the reverse order. When we have no moon on the earth, the earth will appear fully illuminated from the moon; and when we have full moon, the earth will shed no light on the moon. In addition to these changes in the illumination of the earth as seen from the moon, we would also observe that the brightness of the illuminated part of the earth would vary from time to time, and from point to point. On some days more light would be reflected from the earth to the moon than on other days, giving rise to bright days and dark days. These changes would be produced by the changes in our atmosphere, as it is very obvious that when our atmosphere is covered with clouds more light will be reflected to the moon than when the air is clear and cloudless, as clouds reflect more light than either land or sea. From these considerations we see that the brightness of the dark body of the moon is mainly determined—other things being equal—by the amount of cloud in our atmosphere.

The dark body of the moon being visible, or, as it is generally expressed, "the old moon seen in the arms of the new," is one of our well known and oldest indications of coming bad weather. If the explanation given above of the illumination of the dark side of the moon is correct, then it gives an explanation of the old saying, and we see that this lunar indication, unlike many others, has a sound physical basis. "The old moon seen in the arms of the new" indicates that to the west of us there are at the time vast areas of clouds. Now that is the direction from which we receive most of our weather, and the probability is that many of these clouds are over the North-Atlantic Ocean, and will travel eastwards and northwards, and unburden themselves over the north-west of Europe.

If these conclusions are correct, we might look upon the dark side of the moon as an outlying signal station, capable of giving us indications of the more or less cloudy condition of the earth's atmosphere, and with properly constructed instruments, the signals might, with some practice, come to be perfectly intelligible and accurate. There is, however, one unfortunate circumstance which will make the readings of these signals at certain times extremely difficult. The brightness of the dark side of the moon will not only be determined by the more or less cloudy condition of our atmosphere, but also by the amount of the illuminated side of the earth turned towards the moon, and, as this varies from day to day, the interpretation of the signals will be difficult and probably impossible when the moon approaches the full, as the illuminated part of the earth then turned towards the moon is very small. On the other hand, the diminished illuminated area of the earth turned towards the moon, makes the lunar indications in another way more definite, as the moon then reports the cloudy condition of a much narrower area, east and west, of the earth's surface. Allowance would, of course, require to be made for the more or less clearness of our atmosphere.

If we wish to get the amount of cloud during the last half of the moon, we can observe it as during the first half; but now it will give us the cloudiness to the east and not to the west of us as before. So that if we wish to know the condition of the atmosphere to the west of us over the Atlantics, during the last days of the moon, we must have the observations made and telegraphed to us from America.

These suggestions are here offered in an extremely crude state; but as they appear to have a germ of truth in them, they are offered in the hope that some one with the proper means of observation may take them up and put them to a practical test.

2. The Acids of Opium. By D. B. Dott.

The acids which have been described as existing in opium are meconic, sulphuric, lactic, and acetic acids. It is not certain that the two latter are always present. In any case they exist in small amount, and are of little importance. For many years after its discovery in 1805 by Sertürner, meconic acid was regarded not only as the peculiar acid of opium, but as that with which the morphine and some of the other bases are wholly combined. In later years it has become known that the morphine exists partly as sulphate, but I do not find that any analyses have been made, showing to what extent this is the case. The subject has indeed received but little attention, as is evident from the fact that in many of the most recent publications it is assumed that the morphine exists naturally entirely as meconate.

In 1878 Dr C. J. H. Warden of the Bengal Staff published an interesting analysis * of Behar opium ash, obtained by preserving the ashes of all the samples analysed at the Government Opium Factory for several years. The chief feature of interest in the

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* *Chemical News*, xxxviii. p. 146.

analysis is the large percentage (23·14) of sulphuric anhydride found, mostly combined with potash. Mr J. Scott, in his *Manual of Opium Husbandry*, gives analyses of poppy plant ash, in which the sulphuric anhydride varies from 5·93 to 10·08 per cent. Mr Scott points out that the larger the amount of sulphuric acid in the plant, the smaller is the yield of morphia :—

SO_3 in Plant Ash.	Morphia in Opium.
5·93	8·66
7·64	6·47
10·08	4·65

Referring to these results, Dr Warden remarks :—"It would be interesting to ascertain whether or not there is a similar correlation between the SO_3 in the opium ash and the amount of morphia contained in the drug. Possibly the SO_3 in opium may be directly proportionate to its richness in morphia."

In order to determine whether this surmise is correct, the sulphuric anhydride in a number of opiums was estimated by exhausting the drug with water, adding large excess of hydrochloric acid, and then precipitating with chloride of barium. It was found that this method avoided the precipitation of meconate. In the table below are given the percentages of sulphuric anhydride and morphia found in various samples of Turkey and Persian opium. As it cannot be pretended that any process for assaying opium gives absolutely accurate results, the figures under *morphia* are only considered comparative, and as an approximation to the truth.

	SO_3 per cent.	MH_2O per cent.
a,	2·08	8·5
b,	2·01	15·3
c,	1·97	12·3
d,	1·94	12·1
e,	1·74	9·8
f,	1·72	12·2
g,	1·65	7·9
h,	1·50	9·5
j,	1·49	11·1
k,	1·07	2·0

It is evidently impossible to deduce from these results any general law relating the proportions of sulphuric acid and morphia. The most that can be said is that if an opium is rich in sulphuric acid it will probably be rich in morphine; but the determination of the amount of sulphuric acid could not be used as a means of ascertaining the value of the opium. The interesting fact revealed by these estimations is the large proportion of sulphuric acid existing in opium. So far as I am aware, this has not been previously pointed out. It becomes important to know in what manner the acid is combined. In order to determine this we must know what bases and acids exist in opium. The only alkaloids that need be taken into account are morphine, codeine, thebaine, papaverine, narcotine, and narceine, and of these only codeine and thebaine excel morphine in basic power. The principal mineral base present in opium is, as might be expected, potassium oxide. Biltz, in his careful analyses* of Turkey and of German opium, gives the percentage of potassium sulphate as 2·0 to 2·5. By exhausting fine Turkey opium with water and incinerating the extract, I obtained 2·50 per cent. of ash, which indicated 0·95 $SO_3 = 2·06$ K_2SO_4. This confirms Biltz's results. We may safely leave out of consideration all acids except sulphuric and meconic. It might almost be assumed that sulphuric would replace meconic acid in any of the latter's combinations, but to make quite sure, I dissolved a weighed quantity of morphia meconate in water, added an equivalent of standard sulphuric acid, and gently evaporated. When the solution became filled with crystals, these were examined, and found to consist entirely of sulphate. Taking then, the sample (*b*) in the table above given, we find that it yields 15·3 per cent. of morphia hydrate and 2·01 SO_3. Of this amount 0·95 per cent. is in combination with potash and 0·13 per cent. in combination with the stronger alkaloids (as nearly as can be estimated), leaving 0·93 to unite with the morphia. This amount of anhydride is equivalent to 7·04 per cent. of morphia hydrate, or *nearly half the morphia in the opium.* It would be confirmatory evidence if the proportion of meconic acid in opium were found insufficient to neutralise the morphia. This fact is proved by the following experiments:—

(1) 100 grs. of opium, indicating 11·1 per cent. of morphia, were exhausted with water, slight excess of ammonia added, and the solu-

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* Buchner's *Repertorium*, xxxix.

tion filtered. The filtrate was rendered faintly acid with acetic acid, and then excess of barium nitrate added. The precipitate, after being washed with the minimum of cold distilled water, was dried in the water-bath, and found to weigh 10·31 grs. This was treated with nitric acid to remove the meconate, and then ignited, leaving 4·36 grs. $BaSO_4$. The 5·95 grs. meconate of barium considered as $C_7H_2O_7Ba.H_2O$ is $= 3{\cdot}37$ grs. $C_7H_4O_7 = 8{\cdot}0$ grs. morphia.

(2) From a quantity of good opium the morphia and meconic acid were prepared with as little loss as possible. There were obtained 627 grs. morphia hydrate and 186 grs. crystallised meconic acid. The proportion demanded to form the normal meconate is 262 grs. Experiments with other opiums were conducted as in (1), with similar results, the proportion of meconic acid to morphia being even less than that there described.

It is evident that if the morphia salts could be made to crystallise from a simple extract of opium, much light would be thrown on the matter under discussion. With this end in view, 500 grs. of opium were exhausted with alcohol, the alcohol driven off, and the extract treated with water. The aqueous extract was digested with purified animal charcoal, and then concentrated. After some days, a small quantity of crystals had appeared. These consisted of morphia sulphate. (It should be noted here that while the neutral meconate readily crystallises, the acid meconate has never been obtained in the crystalline state.) This cannot be regarded as an altogether satisfactory experiment, on account of the small yield of sulphate, but considering that the extract is charged with substances which hinder crystallisation, the result is not surprising; the use of any purifying agents which would cause decomposition being, of course, quite inadmissible.

To thoroughly elucidate the question would require an enormous number of experiments; but judging from the facts ascertained, I am of opinion that morphia exists in opium, partly as neutral sulphate and partly as acid meconate.

3. Direct Observations of the Effect of Pressure on the Maximum Density-Point of Water. By Professor Tait.

Monday, 4th June 1883.

MR ROBERT GRAY, Vice-President, in the Chair.

The following Communications were read :—

1. The Diurnal Oscillations of the Barometer. Part II. By Mr A. Buchan.

2. Ninth Report of the Boulder Committee. Communicated by Mr Milne Home.

I.—*NOTES BY CONVENER.*

ARGYLESHIRE.

I. 12*th July* 1882, *Stonefield House*, *Argyleshire*, residence of C. G. Campbell, Esq.—Was guided by Mr Alexander of Lochgilphead, to the hills of *Glen Ralloch*, situated to the north of the narrow neck of land which connects East and West Loch Tarbert. The rocks of the hills are gneiss, full of quartz veins. When among those hills, I saw many boulders of small size lying on the sides, and some on the very tops. Their composition, resembling clay-slate, differed from the rocks, and they were all more or less angular. They were mostly on slopes facing, or exposed to, westerly points.

On reaching a hill on the north side of West Loch Tarbert, and sloping down due south towards the loch, at an angle of about 40°, and at a height above the sea of 400 feet, fell in with a boulder lying on the surface, $7 \times 5\frac{1}{2} \times 3$ feet. The rock, visible almost directly under, and at all events very close to, the boulder, was a schistose clay-slate, but with a sprinkling of gravel over its outcrop. The boulder could not probably come from the north, as the hill in that direction rises to a height of about 200 feet above the boulder, and is steeper near the top. The directions from which the boulder might most easily have come are S.E., S., or S.W., this last being the line of the arm of the sea called West Loch Tarbert. The hills to the west, and still more to N.W., appeared too high to have allowed the boulder to have come across them.

II. 14*th July* 1882, *Ormsary House* (on south bank of Loch Killesport), *Argyleshire*, residence of Mrs Campbell.—Set out, under the guidance of Mr Alexander, to visit *Clach Briach* (stone-spotted hill), on which he informed me I would find a number of large boulders.

This hill being situated a few miles to the east of Ormsary House we had to pass the "*Big Boulder*" near the high road, described in a previous Report (Sixth, p. 14), and illustrated there by diagram 5.

I was again struck with the fact, that at this spot there is a great multitude of boulders, several of them touching one another. I counted ten, occupying a space less than 2 acres in extent; one of these (apparently not mentioned in the previous Reports) measured 16 × 12 × 8 feet.

Mr Alexander informed me that on the hills along the south side of Killesport, *west of Ormsary House*, there are no boulders; and that they occur only on the hills to the east of Ormsary House, with the exception of two on the sea-beach. The only peculiarity which I discovered in these respective hills was that to the west of Ormsary the sides of the hills facing Loch Killesport are excessively steep, whereas the hills to the east of Ormsary slope more gently to the Loch, and are not so high. If the boulders were brought on floating ice from the W., or W.N.W., would the last-mentioned hills, because of their more gentle slope, not have more readily arrested the ice, and have afforded sites for boulders when the ice melted?

We passed through a valley called *Baronlungart*, running E. and W. between *Ormsary* and *Achoos*. In the bottom of the valley there are several spots where the rocks are beautifully ground down and smoothed, evidently from the westward. This valley is about 60 feet above the sea-level. A few boulders are lying in the valley.

Having reached the shepherd's house, on *Clach Briach* hill, I mounted a horse and followed a peat road for about a mile in a westerly direction, till we reached a level of about 400 feet above the sea, and came to a place from which we could look down to the north on the farmhouse of *Tign-a-Kaim*. The hill, along the ridge of which we had ascended, terminated at this place in a rounded end, sloping down northward, westward, and southward. The slope most thickly covered with boulders was that sloping to N.W. at an angle of about 40° (see diagram 1). The three largest were of the following sizes:—15 × 8 × 5 feet; 18 × 9 × 8 feet; 12 × 7 × 4 feet.

The boulders are all, more or less, well rounded. The sides most rounded were those facing N.W., suggesting the idea that, after having reached the hill, they had been exposed for some time to friction from some agent impinging on, or passing over, them.

They appeared to be all composed of one description of rock, viz., a compact fine-grained gneiss, which is also the composition of the Ormsary "Big Boulder" and its companions, before referred to. The rocks *in situ* on this *Tign-a-Kaim* hill are soft schist, and on edge.

On the highest part of the hill, and about 20 or 30 yards on the south side (at A on fig. 1), there are several boulders in positions of considerable interest. These are shown on fig. 2. Where boulder A is represented on fig. 2, the ground is nearly flat; at B, the ground begins to slope slightly down south; and at D, the southward slope is as much as 20° or 23°. Boulder D has a girth of about 26 paces, or 78 feet. Its height is about 15 feet. The size of B is 10 × 10 × 10 feet, and of A, 6 × 5 × 3 feet.

It was observed that a fragment had been broken off each of the two largest boulders at their south ends. The form of the fragments and their proximity to the boulders made this evident. There may originally have been cracks in the boulders, allowing rain to enter, and the action of frost to split off the ends. Another conjecture is, that if the boulders, when brought to the hill, fell from any height, and if they had a projecting piece of rock at their south ends, the concussion in the mass, produced by the central solid portion of the boulder first striking the hill, might cause the projecting piece to break off. The direction of the longer axis of the largest boulder D is about W.N.W. and E.S.E.

Not far from these, and also on or close to the highest part of the hill, there are other two boulders (shown on diagram 3) touching one another, A being 17 × 8 × 8 feet, and B 18 × 10 × 10 feet. The direction of the longer axis of A is S.W., and of B, N.W. A small boulder lies between the two, at the north end, firmly jammed. It seemed probable, from the positions of the boulders, that boulder B was the first to come, and that the others subsequently had been intercepted by B in their further progress eastward.

From this hill the three Paps of Jura, 2500 feet high, are visible, bearing W.N.W. The more distant island of Mull, reaching to a height of 3000 feet, bears N.N.W. If the boulders on Loch Killesport, above described, came from either of these sources, they must have crossed, not only two sounds or arms of the sea, but also the two or three tongues of land which project from this part of the coast of Argyle. Before, however, that suggestion can be favoured, it would require to be shown that the rocks composing the boulders are similar in composition to rocks in Jura or Mull.

To the west of the *Tign-na-Kain* hills, above described, there is another hill, separated from them, by a small valley, on which many boulders are visible, on the side of the hill sloping to the north. This hill is almost 100 feet higher; but I could not ascend it, much to my regret.

18*th July* 1882, *Taynish House*, on Loch Sweyn, the property of Captain Campbell of Inverneil.—The house, garden, and policy are at the point of a narrow tongue of land, on the south side of *Loch Sweyn*, which projects into the loch in a S.S.W. direction. Near the point where this tongue reaches the sea there is on it a ridge, running (by compass) about W. by S., with a breadth of about 100 yards, and rising eastwards to a height of about 100 feet above the sea.

This tongue of land, especially on its higher parts and on its northern slope, is covered by above 50 boulders, the largest of which are of the following sizes:—

18 × 11 × 8 feet. Its longer axis lies W. by S., and small end to west. It lies on the broken edges of vertical strata, as shown in diagram 4. Its longitudinal axis dips to west, at an angle of about 20°. *r*, *r*, *r* are edges of the rocks *in situ* on which the boulder rests.

15 × 19 × 5 feet. Its longer axis is N.E. and S.W.

8 × 4 × 3 feet. „ W. by S.

8 × 4 × 2½ „ W. by S. with small end to westward.

9 × 6 × 4 feet. Its longer axis W. by S.

Diagram 5 is a ground plan of the surface on which these boulders lie,—*b* is about 100 feet above the sea, and *d* about 60 feet; *b*, *d* representing a ridge running about W. by S. and E. by N., with the surface sloping gently down on each side towards the north and south respectively, as shown by arrows and the letters *e*, *f*. Much of this rocky tongue on which the boulders lie has been ground down to a smooth surface.

The site of the largest boulder B represented is shown in diagram 5 on the northern slope at B.

The greatest number of boulders deposited is on the northern slope (diagram *e*, *a*, *b*), as if they had come from some north-westward point, and had been intercepted by the slope.

The position of these boulders, especially of the largest, renders it more probable that they came by floating ice from the sea, than by a glacier from the land.

Learnt from the shepherd's wife, an intelligent woman, residing in the offices of Taynish House, that there are two other large boulders at or near the shore, about 300 yards south of Taynish House; but bad weather prevented a visit to them. In the policy of Taynish, close to the avenue, about half a mile to the east of the house, we observed many boulders on a hill side, sloping down to the north, at about 200 feet above Loch Sweyn.

On our return to Ardrishaig I visited again the large boulder at *Loch Mhurrich*, mentioned in the Sixth Report, p. 16. Ascertained that the depth or vertical thickness of the boulder was, at its east end, 12 feet, and at its west end, 5 feet; its narrowest end being therefore towards the west. Its longer axis, which is W.S.W. and E.N.E., slopes down towards the west.

The situation of this boulder, relatively to the adjoining hills, is shown in diagram 6, where B is the boulder and H rocky hills surrounding the valley, with small boulders scattered over them, shown by dots. These hills rise to the height of from 200 to 300 feet above the sea.

When looking from the boulder towards the west, a range of low hills is seen crossing the valley, about half a mile distant, and in that range a depression occurs, through which the road passes, leading westward to Keills. The summit level of this depression is about 100 feet above the sea, whilst the rest of the range crossing the valley reaches to a height of from 300 to 350 feet. The bearing of this depression from the boulder is W.S.W., coinciding with the direction of the longer axis of the boulder. If the boulder had been carried to the spot where it now lies, from the westward, it would probably be by ice floating through the depression before referred to. For any land glacier the locality seems quite unsuitable.

On our way back to Ardrishaig we observed, on the hills within sight of the road, many boulders. They lie most frequently on slopes facing some westerly point. Thus, on a hill called "*Leck-na-Ban*," on the north side of the road, at about 300 feet above the sea, where the slope is towards W.S.W. at an angle of about 10°, a boulder 10 × 8 × 6 feet is lying on the edge of vertical strata, the boulder being a light-coloured fine-grained crystalline gneiss, while the rocks on which it lies are a soft slaty schist.

Near the Crinan Canal at *Ballanoch*, on the hill above the high road, and about half a mile from the canal, there is a boulder 16 × 9 × 9 feet, at a height of about 300 feet above the sea. It lies on the north side of the valley through which the road passes. The general direction of the valley is N.E. by N. and S.W. by S. The longest axis of the boulder coincides with the direction of the valley. The boulder is lying on bared rocks. It seemed probable that ice carrying boulders had floated through this valley, and lodged the boulder.

19*th July* 1882, *Ardrishaig Hotel.*—In the Seventh Boulder Report, p. 10, a very partial account was given of smoothed and striated rocks at *Kilmory*, at the western extremity of the tongue of land dividing *Loch Killesport* from *Loch Sweyn.* I therefore returned there, to examine the spot more minutely, in company with Mr Alexander of Lochgilphead.

At *Ardna*, Mr Macmillan's farm, I saw again the large expanse of these interesting rocks.

The extent of rock surface, horizontally, is about 13 yards, and vertically, about 5 yards. The surface in different parts slopes down towards S. by E.—S.—S.S.E.—and S.E., at angles varying from 30° to 40°. They have been most severely rutted, on the slopes which face S. and S. by E. Where the surface slopes down S.E. it is not striated, only smoothed; showing that the striating agent did not move in such a direction, as to touch or strike, or at all events press severely on the S.E. slope. In some places the striæ were seen to have been more deeply cut at their *west* ends than at their east ends. Some of the striæ at their west ends are as much as 3 inches wide. The direction of the striating agent must therefore have probably been from W. by S., or due west, to have made the striæ. Portions of the smoothed rock surfaces were broken into small shallow depressions, and in these pebbles of hard rocks were observed, somewhat firmly packed, and where probably they have been lying since the time they were originally deposited. It was by such tools as these that the striæ had no doubt been formed on the smoothed surfaces of the rocks. A representation of a few of these striæ, and of the depressions in the rocky surface, is given in diagram 7.

That there must have been heavy pressure on these smoothed rocks, is evident from this fact, that though the rocks are dipping or sloping down towards the south, at an angle of as much as 40°, the striæ are all *horizontal*, or nearly so,—showing that the striating body was of such bulk and weight as to keep steadily on in its course, in spite of the tendency, by gravitation, to slide down the face of the rock.

On the hill where these smoothed and striated rocks occur, boulders of small size (comparatively), and much drift of hard rounded pebbles, are plentiful.

After examining these rocks I climbed the hill to the eastward to a height of about 600 feet, and passed several striated rocks, and three boulders of the following sizes:—11 × 7 × 3 feet, 12 × 5 × 2½ feet, 15 × 6 × 4 feet. Each boulder has its longer axis pointing in the same direction, viz., W.S.W. and E.N.E. These are situated near the top, and on the side of the hill sloping down towards the S.S.E.

Having crossed the ridge of the hill towards the north, and descended a little way on the side sloping down towards N.N.W., I was struck at finding almost all the boulders lying with their longer axis W.N.W. and E.S.E. The following are the sizes of the largest boulders examined:—11 × 6 × 3 feet (with sharp end to N.W.), 14 × 8 × 3 feet, 14 × 7 × 7 feet (its longer axis was W.S.W.), 21 × 7 × 3 feet, 9 × 6 × 4 feet (its longer axis due west); but here there was a change in the down slope of the hill, viz., towards W. by N., instead of N.N.W. This last mentioned boulder was lying not on drift, as the others were, but on bare rock.

A little further east, where the high road passes through the lands of Castle Sweyn, but above the road, and at a level of about 200 feet above the sea, found a group or cluster of boulders, four or five in number, touching and partly covering one another, as shown in diagram 8. The remarkable feature of the spot is, that the slope of the hill here is so steep—about 40°—that it was hardly conceivable how the blocks should, when laid down on such a slope, have remained on it, and have for ages retained their position. The only probable explanation seemed to be, that beneath the two lowest boulders there were portions of projecting rock which supported the whole group. The slope of the hill here is down towards W.N.W. A study of the whole position led to the conclusion that these boulders, to obtain their lodgment, must have been brought from W. by N.

Still farther east the road passes over an extensive plateau or table-land of drift, which has a general height of 120 feet above the sea. The farm of *Doidhe* is here. A gravel pit (almost 8 feet deep) for the excavation of road metal was examined. The layers of gravel and sand in it were found to be horizontal.

All along the road towards *Ashfield*, *Deltot*, and *Achnamara*, large boulders occur on both sides, though not in nearly such numbers, as near the open sea at *Kilmory*, and near *Castle Sweyn*.

20*th July*, *Ardrishaig.*—Was guided by Mr Alexander to the farm of *Ach-na-Brack* (*Field of Spots*) to see some remarkable sculptured cup-markings on smoothed rocks.

These rocks occupy an extensive portion of pasture ground. They are of hard gneiss, and the smoothed surfaces slope down towards about S.W. at an angle of 10° or 12°. They have evidently been smoothed by natural agency. Striæ occur on them at several places. The direction of the striæ varies a little, being in some spots from W. by N., in others from N.W. One small boulder was seen, on the west side of one of the smoothed ridges of rock, and seemed to have been stopped by the rock in its progress eastward.

The cup-markings are very numerous, and consist as usual of circular ruts as shown in fig. 9. They are of different sizes; the largest about 2 feet across. The straight rut issuing from the centre, and cutting across the circular ruts, had in almost all cases been formed in unison with the downward slope of the rock. There are 20 or 30 of those cup-markings, and they well deserve to be described and sketched. Archæologists have never yet been able to suggest any plausible explanation of the object or meaning of these ancient symbols.

BERWICKSHIRE.

August 1882.—Convener received from his factor, Mr Muirhead, a chip of a small dark-coloured syenite boulder, found on Lamberton Hill, four miles north of Berwick, at a height of about 600 feet above the sea. It was found near the summit of the hill, which there forms a ridge running N. and S. It was on the slope of the hill facing the west. The only locality in Berwickshire for syenite rock is Stenchel Hill, on east side of Cockburn Law, distant about 10 miles to W.N.W. The size of the boulder was 30 × 16 × 16 inches, weighing about 20 stones. The whole of Lamberton Hill is a mass of porphyry.

II.—*NOTES BY PROFESSOR HEDDLE.*

1. *Excursion from Killin (Perthshire) up the N. and S. valley of Radour, in Kenmore Parish,—over Heasgarnich,* 3530 *feet, to Loch Lyon,—and thence down the Allt Chonoghlais valley to Tyndrum, accompanied by Rev. Mr Peyton, of Free St Luke's, Broughty Ferry.*

1. In Radour valley is found on the slopes of *Creag nam Bodach*, at altitude of 1400 feet above the sea, a boulder 9 × 7 × 7 feet, of "pure *white quartz*," sharp and angular. The rock of all the hills hereabouts is "*flaggy gneiss.*"

There is such quartz rock on *Creag Mhor* and *Ben Dorean*, situated to the W. and N.W.; also, a quartz rock, though not quite similar, on *Meall Ghaordie* to the east.

2. On the *Heasgarnich* side of the same valley, there is a boulder 13 × 5 × 6½ feet, somewhat more gneissose than the flaggy rock of the district, at an altitude of 1600 feet.

3. Towards the rounded head of the glen, at an altitude of about

1650 feet, there are eight other boulders, averaging 11 × 5 × 4 feet, and forming a line across the valley.

4. The pass at the head of *Loch Lyon* is crowded with *till*, which fills great stretches of the valley. It has been by some means shaped or worn into conical mounds, and abounds from the level of the lake, at 1100 feet above the sea, down to about 800 feet.

2. *Excursion from Killin to Bowachter, in Glen Dochart (Perthshire), and thence over Sgiath Chrom,* 2780 *feet; Sgiath Chuil,* 3050 *feet; Meall Chuirn,* 3057 *feet; and along the ridge over Meall na Saone,* 2835 *feet, eastward to Mid Hill,* 1977 *feet; accompanied by Professor Butler and Mr Colin Phillip.*

1. On the south side of *Sgiath Chrom*, at altitude of 1520 feet, found two boulders; one in size about a cubic yard, the other 8 × 5 × 4 feet. This last was of hornblendic gneiss with chlorite, similar to a rock seen by me on a previous occasion, between *Ben Laoigh* and *Ben Oss*, situated about 10 miles to W.S.W.

2. On the east side of *Meall Chuirn*, found a valley or trench running N. and E., about 750 feet deep, separating that hill from the ridge to the east. On the east slope of this trench, found some loose rock, apparently not fallen from the upper part or side of the valley, but ice-carried.

3. Proceeding towards the eastward, found three small hills with flattened summits, each over the 2750 contour line, but not named on the Ordnance map. The most northern of these is very precipitous on its sides, and separated from the other two by narrow cols about 80 feet deep. From the top of one of these, which we ascended, we descried a boulder perched on the top of another hill with precipitous sides, lying in a slight depression. The boulder was apparently about 10 feet cube. Considering this boulder, on account of its position, to be one of interest, we retraced our steps about 300 yards, to try and reach the boulder, but were defeated. There was over a foot of newly-fallen snow, which prevented our finding a firm footing, and caused constant and dangerous slipping.

4. Proceeding about three miles farther eastward, towards the *Mid Hill*, 1977 feet, we had to cross a depressed flat surface, now a peat bog, about 200 feet below the top of the hill. The summit

of the hill consists of flattened rock, whose contour "shows unmistakably that its form is the result of a body of ice having passed over it from the west." Its eastern end shows marks of having been broken; and some little space eastward from the cliff there is a great rugged block, which seems to have been detached from the rocky cliff, and pushed a little way eastward.

3. *Excursion from Luib Railway Station northward, over Beinn nan Clach,* 2309 *feet; Bein nan Imirean,* 2500 *feet; Bein Glas,* 3139 *feet; Bein Dheiceach,* 3074 *feet; then down stream-valley, back to Luib;—with Mr Colin Phillip.*

1. Found the whole south side of *Beinn nan Clach* from 2000 to 2100 feet, sprinkled with boulders from 1 to 3 cubic yards in bulk. They consisted of a kind of rock differing from that of the hill, being more chloritic.

Found a line of somewhat larger boulders lying along the top ridge of the hill, stretching in a line towards *Imirean.* Found a much rounded block about two cubic yards in bulk upon the *solid rock of the very summit of the hill,* at a height of 2309 feet above the sea (see fig. 10). The rock of the summit was also much rounded.

Noticed also the outcrop of some nearly horizontal strata, with scattered dislodged fragments; suggesting the action of some moving body which had been grinding on and rupturing the edges of the strata.

Found blocks at about the same, or rather lower level, in the corry between *Beinn nan Clach* and *Beinn Glass,* as also at the foot of the east slope of *Bein Dheiceach;* but these might have fallen from rocky cliffs above them.

During the two last excursions, looking across Glen Dochart southwards, some very large boulders were descried on the N.E. shoulder of Ben More (3843 feet). They were on a little flat, at about from 1750 to 2000 feet up. These could not be "fallen blocks." The ridge on which they rest was too narrow, and the slope too great, for falling masses to have been arrested where they lie. Ice seemed the more probable agency.

On a review of the facts observed during the foregoing excursions, I cannot explain them by the agency of local glaciers; nor can I conceive how a great solid mantle of ice covering the whole country, and sliding over the hills, could have left the number of blocks we saw on their summits.

4. *Excursion over the hills situated to the east of Loch Laggan, in Lochaber, Inverness-shire.* (IV. *and* V. *were with Rev. Mr Peyton.*)

Started from Loch Laggan Inn, and went N.E. by *Meall Ghrealach* (1650 feet), *Buidh' Aonach* (3037 feet), along the ridge to *Creag Meaghaidh* (3700 feet), thence descending upon Moy.

On the south slope of *Buidh Aonach*, two grey granite boulders were found on a small plat at 2260 feet contour line.

About the centre of the long ridge, and nearly due N. of the centre of Loch Laggan, there is a round and broad eminence 3238 feet high. This eminence is bedded with gravelly clay, resulting apparently from the disintegration of granite belts in the gneiss. We counted on this eminence about twenty-four large and much-rounded grey granite boulders, identical in character with those seen on the south slope of the hill above mentioned. Some of them were lying on the surface of the clay, some were half bedded. Veins of granite were found in the flaggy gneiss of the hill. In these veins there were masses in every stage of being rounded by decay and by weathering, becoming loosened out of the gneiss rock, with portions of the rock adherent. None were so large as the blocks lying in the adjoining district.

It occurred to me that many grey granite blocks considered to be boulders may have originated in this way, and máy have been pushed from their birthplace,—as, for example, the blocks seen on the south slope of the hill above referred to.

In last year's Report reference was made to grey granite boulders seen on or near the top of *Craig Dhu* (2161), at the mouth of Glen Roy, about 15 miles west of Buidh Aonach. Might these not have originated in the same way? No veins, however, were seen on Craig Dhu.

5. *District near Loch Clunie, north of Caledonian Canal.*

1. In driving up *Glen Morriston*, we observed at the east end of the summit of a hill about 1000 feet above the sea, near the junction

of the River Loyne with the stream flowing from Loch Clunie, "a *grand* boulder;" but it was some distance from the road along which we were driving, and we could not reach it.

From Clunie Inn we ascended *Carn Ghluasaid* (3140 feet); *Carn Glas* (3260 feet); *Sgùrr nan Conbhairean* ("*Ben-doe*") (3634 feet); *Carn Dubh Liath* (3280 feet); and *Garbh leac* (3673 feet), and then back to Clunie Inn.

On the south slope of the first-named hill, at a height of 1475 feet, we found two grey granite boulders, each about 3 cubic yards in bulk, stopped against a little knoll to the S.E.

3. On another day we climbed *Stob Bathaich* (2740 feet) (opposite *Am Bathaich*), the S.E. spur of *Carn Fuaralach* (3241), and found on it two blocks. At the height of 850 feet, and at a distance of about 400 yards north of Clunie Inn, several fragments of rock were found, which we ultimately considered to be fallen rocks.

At a height of about 1520 feet, on the east slope of the same hill, we found several large blocks, which had been clearly transported. They are angular, and their resting on so steep a slope was most striking. The largest is represented on fig. 11.

On the same hill, at a height of about 2000 feet, and on its S.S.W. slope, we found an angular block 13 × 5 × 4 feet. Observing not far off an outcrop of rock of the same description (white quartzy gneiss), about 7 feet high, and bearing W.N.W., about 65 paces distant, and at a somewhat higher level, we saw such appearances on it as to convince us that the block had somehow been torn off from this rock, and lodged where it now lies, viz., about 15 feet below the level of the rock. Between the block and the outcropping rock there is a gully, or hollow, about 15 feet deep, now occupied by a small stream, across which the block must have been carried,—by what agency is the question (see diagram 11).

3. We next proceeded over the hill tops the whole way north to Achnasheen, and were much interested in the number of cases of boulders lying on very steep slopes of high hills. One of the hills ascended was *Sgurr na Lapaich* (3778 feet). From the spongy nature of the grass it was the hardest climb I ever experienced. For about 1500 feet above *Loch Mullardoch*, the slope was at an angle of 47°. At the height of about 1530 feet (above the sea) there rests on the slope a boulder 12 × $8\frac{1}{2}$ × $7\frac{1}{2}$ feet, of hard quartzy

gneiss. "Did that stone roll down that slope and stick there?" I asked the shepherd who walked with us. The instantaneousness and energy of his "Never!" was delightful. "And why not?" I asked. "Well, in the *first* place, not a peeble can stop on sic a slope, let alane sic a stane as yon, if she ance fetched wey; in the *second* place, there are nae stanes ava on the tap, but jist a peat bog; and in the *third*, there's no a stane just exactly of this natur' in the hill." We found, on surmounting the first heave of the hill, 2250 feet, a wide expanse of bog.

6. *Aberfoil.*

Ascended two small hills. The first (*Arndrum*) is a portion of the ridge of *conglomerate rock* which, in the east, occurs at Callander, and in the west crosses Loch Lomond, forming a line of islands. The ridge thus stretches E.N.E. and W.S.W.

On this hill I found, at the height of 230 feet, a line of six boulders of angular fragmentary gneiss (greywacke), stretching from N. to S. They were closely adjacent, viz., from 2 to 20 feet apart, and from $\frac{3}{4}$ to 3 cubic yards in size.

To the west of this line, four other similar boulders lay along the summit of the ridge, and thus at right angles to the first line. They stretched nearly to the top of the hill, viz., to 454 feet.

III.—*NOTES BY MR MURRAY, GLASGOW.*

ISLANDS OF COLONSAY AND ORONSAY.

Mr Murray of No. 169 West George Street, Glasgow, having during the last two summers spent some weeks in the island of Oronsay, was so obliging, at the request of the Convener, as to search for boulders on these islands, and has sent to the Convener notes, from which the following are extracts:—

1. *Oronsay.*

Along the sea-beaches the shingle is found to contain pebbles and blocks of syenite, grey granite, and greenstone.

The syenite may probably have been derived from rocks at Kiloran Bay, in Colonsay, about 9 miles distant to the N.N.W.; and the grey granite and greenstone from rocks at and near Schallasaig, about 5 miles distant, on the east side of Colonsay, to the N.N.E.

There are also fragments of a bright red granite, with large crystals; but no rocks of a similar composition were met with on either island. These were found at "Port na Long," on the west shore of Oronsay, and also at Poll Gorm, on the eastern extremity of Oronsay.

A little to the south of the large sandy bay on the east side of Oronsay (Traigh na Shella), there is a boulder of coarse-grained granite, "of a *pinky* colour," probably the same kind of rock as that found at and near Schallasaig. Its size is 3 × 2 × 2 feet.*

Near this boulder there is one of quartzite, 2 × 2 × 2 feet, differing in composition from any rocks seen. Beside it there are nodules of chocolate red sandstone; also not seen on the island *in situ.*

A little to the south in a narrow gully, there is another coarse quartzite 4 × 3 × 3 feet, partly bedded in the sand.

Highest hills in Oronsay (304 feet) are rounded, and the rocks near the base are generally smooth, like those now washed by the sea. No boulders were seen on them.

The strand, a low sandy tract, dividing Oronsay from Colonsay (covered by the sea at high water), has several boulders of small size on it, and in particular one of red sandstone and one of a reddish-grey granite.

On the west side of the strand, on the farm of *Garbh*, there are several small boulders of grey granite.

2. *Colonsay.*

At Schallasaig (a harbour on the east coast) there is a considerable extent of grey granite rocks *in situ*—and across the island, towards the west, there is a depression or hollow, which reaches to the coast. Strewed over this hollow there are blocks of Schallasaig granite. About half-way across there is a hill about 20 feet high, called "*Cnoc an Ard Righ.*" The Schallasaig granite rocks apparently come no farther west than this hill.

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* *Note by Convener.*—Professor Geikie, in a recent paper published in the *Transactions of the Glasgow Geological Society* (vol. vi. p. 160) on the "Geology of Colonsay," mentions, that in the neighbourhood of Schallasaig there is a "granitoid rock" containing *felspar*, "which sometimes shows a rosy flush."

Near Meall Buic, situated about 2 miles S.W. of Schallasaig, a great pit has been opened for gravel and sand. In this pit, a number of boulders occur, the largest about 4 feet long. Two of these have striæ or ruts on their surface, parallel with longer axis. Though many are apparently of Schallasaig granite, there are others, which were not recognised as the same as any rocks on the island.

Some basaltic or whinstone dykes occur on the east coast, a little south of Schallasaig; and boulders of these occur to the west.

At the foot of Dungallon hill (on S.W. coast) there is a sand pit, with a considerable number of boulders, from 1 to 2 feet long, apparently water-rolled. The height above the sea is from 40 to 50 feet.

On the south side of Dungallon, there is Port Loth, where several boulders (about 2 × 2 × 1 feet) of grey granite occur. In a gully in the rocks on N.E. side there is a grey granite boulder, 4 × 2½ × 2½, and one of trap, 4 × 3 × 2½ feet, well smoothed. To the north there are more, all apparently the same as Schallasaig; but besides them, there are some granites of a yellowish-red colour, different from any *rocks* seen.

In the N.W. part of Skipness the hills are rounded and smoothed, but no boulders were seen on them.*

3. *Shingle Beaches.*

Along the shores of both islands, and even on inland spots, especially in Colonsay, there are extensive collections of pebbles and small boulders, all evidently water worn,—some of them reaching to a height of 70 feet above the present shore. The pebbles and boulders consist chiefly of hard crystalline rocks—granite of the grey variety is the most common, but there is also red granite and syenite. One specimen of *flint* was found, much rounded and smooth, as if water worn. The specimen having been sent to the Convener, he forwarded it to Professor Heddle for his opinion. He returned it, stating that it is a "chalcedonic silica," not in the least resembling what occurs in "the Liassic beds of our west coasts," and he suggests that it had probably somehow been transported from Ireland.

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* *Note by Convener.*—With reference to the granite boulders, which in most cases Mr Murray seems inclined to connect with the Schallasaig granite rocks, it should be kept in view that at Killoran (in the N.W. of Colonsay) there are, as Mr Murray states, "rocks of both red granite and grey granite, which rise in masses to the height of 200 feet. The grey is found as far east as Ballinahard. At the same place there is an exposure of a third granite, of a dark colour, resembling syenite. In the same neighbourhood the schist is much contorted and burnt; and at its junction with the granite there is a vein of quartz rock or quartzite."

Professor Geikie, in his memoir on the "Geology of Colonsay," already quoted confirms Mr Murray's statement, when he refers to "a crystalline rock which appears to be of igneous origin, a syenite consisting of pink felspar and dark green hornblende. It occurs near Skipness, in the north of Colonsay, in which district, the schistose and gneissose strata are much broken and confused."

The Convener asked whether this flint specimen may not have been brought by a boat or ship? Mr Murray replied that this was impossible, on account of the position where it was found.

Mr Murray gives a list of the various kinds of pebbles and blocks found by him in the raised shingle beach on this part of the coast, and he has sent chips of these with his notes.

In this list the following are mentioned:—

"Red granite, with large flakes of red felspar.

"Claystone porphyry, with imbedded crystals of a chocolate colour.

"White micaceous indurated sandstone.

"Crystalline slate.

"Indurated sandstone; irony stained.

"Chocolate-coloured claystone.

"Syenite.

"Quartzite.

"Claystone porphyry amygdaloidal; chocolate coloured; large imbedded crystals."*

A little way to the north of Dungallon, the raised shingle beach s extensive. There are three terraces rising from the sea-level, each about 7 feet in height. The uppermost and the one next to it, each forms a horizontal flat, about 10 feet wide. The level of the highest part of the beach is about 80 feet above high-water mark.

The shingle extends as far north as the village of Kilchattan. Traces of an old beach at the height of about 60 or 90 feet above sea-level are visible there.

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* *Note by Convener.*—Can this be a pebble derived from the rocks in Killoran Bay, described by Professor Geikie as "a remarkable volcanic agglomerate—made up of the broken angular débris of the strata by which it is surrounded?" (Page 160.)

IV.—*NOTES BY DR TRAILL (ORKNEY).*

Ronaldshay Island (Orkney).

North Ronaldshay.—In the Committee's Report for 1882 Dr Traill stated that the rocks *in situ* consist almost exclusively of Old Red Sandstone flags. On the surface of the island there are several boulders of foreign origin, viz., one of "*coarse conglomerate*," a rock occurring at Heclabir, in the adjacent island of Sanday, situated to the S.W.; smaller blocks of *granite* and *syenite*, transported possibly from Stromness, Pomona Island, distant from Ronaldshay about 45 miles to S.W., between which and Ronaldshay there are several islands and deep-sea sounds; also a stone resembling coarse *jasper*. Some of these blocks have their surfaces flattened and smooth, and in some instances shining, as if they had been ground down and polished.

In a subsequent communication to the Convener, dated 24th Nov. 1882 Dr Traill states, that having recently had to cut a number of trenches or drains through his property near the centre of the island, he had found boulder-clay containing specimens of *flint*, *chalk*, *oolite*, *limestone*, and *sandstone*, and other stones foreign to the island. A portion of one of these stones having been submitted for examination to Dr Heddle, of St Andrews University, he considered it to be a portion of augitic rock, much the same as what he had seen in the island of St Kilda, but nowhere else.

In the North Ronaldshay boulder clay, fragments of marine shells occur, especially *Cyprina Islandica*, and traces also of the more fragile *Astarte* and *Dentalium*.

Professor Heddle writes (16th Dec. 1882) to the Convener regarding these specimens as follows:—"I send to you one of the pieces of flint which Dr Traill got out of the 'till' of N. Ronaldshay, to show how similar it is to the piece of flint you sent to me from Colonsay, which I have no doubt came out of Irish limestone. I never saw chalcedonic silica (as this is) in the least resembling it, in any of the Liassic beds of our West Coast.

"Dr Traill has also shown to me among his spoils from the North Ronaldshay 'till,' two unquestionable pieces of Torridon conglomerate. There is no manner of doubt as to these. He saw many large pieces of it. These *must* have come from some site between the Kyle of Durness and Loch Carron.

"Again, another piece of rock is not to be distinguished from a mass of granular labradorite, which I found near the summit of one of the hills in Rum Island.

"I have taken some trouble with these specimens, qualitatively analysing about four of them. Though somewhat differing in appearance, they are all coming out a finely crystalline, dense, pale yellow *dolomite,* approaching to marble, and so may have come from Assynt or the north of Ireland.

"It is most marked, the great number of pieces of this one kind of rock. I regard it as a '*find*' of importance; for surely now that it has been analysed, the rock can be found somewhere *in situ.*"

In a memoir on the Orkneys by Messrs Horne and Peach (*Quarterly Journal of London Geological Society* for Nov. 1880), it is mentioned, that in the island of *Stronsa* (not far from Ronaldshay) boulder clay on the eastern shore forms "a continuous cliff for nearly half a mile." "The deposit, which varies from 20 to 30 feet in depth, consists of a tough gritty clay, of a reddish colour, full of well-smoothed and striated stones." "The greater number of included blocks have been derived from the flagstones and sandstone rocks of the neighbourhoods; but the following rocks are likewise represented:—*Granite, pink porphyritic felstone, gneiss, schist, quartzite, white quartz, dark limestone, oolitic breccia, chalk,* and *chalk flints,*—all of which are foreign to the island." (Page 657.)

"Equally important is the presence of numerous *fragments of marine shells* throughout the deposit." "Nearly all the fragments are smoothed and striated, like the stones in boulder clay; there can be little doubt that these characteristics are due to the very same cause in both cases." (Page 657.)

"In the island of *Shapinshay* shelly boulder clay occurs at various localities, and contains finely striated chalk-stones." (Page 657.)

V.—*BOULDERS ON EIGG, AS DESCRIBED IN LETTERS TO THE CONVENER, FROM NORMAN MACPHERSON, ESQ., PROPRIETOR OF THE ISLAND.*

1. *Rocks seen on the Surface.*

1. The S.W. part of the island has on it the remarkable ridge of Pitchstone Porphyry, described by Professor Geikie and others, called *Scoor-Eigg.* The length of the ridge is altogether about two miles. It reaches at its east extremity to a height of about 1300 feet above the sea, at its west extremity to a height of from 900 to 990 feet.

It rises from a plateau, which is about 400 feet above the sea.

Its eastern half runs about E. and W., its western half N.N.W. and N.W.

Both north and south sides are precipitous, almost vertical, showing on the north a cliff 270 feet, and on the south a cliff 400 feet in height.

2. The north part of the island, it is believed, consists of bedded basaltic rock, but, being well covered by grass and moss, the rocks are nowhere visible. The island at its N.E. end rises to a height of about 1080 feet above the sea. At its northernmost extremity it is 990 feet above the sea.

The top of the hill is a smooth plateau, about half a mile broad, from which there is a precipitous dip towards the sea, on the N. and E. It is also precipitous towards a flat basin or hollow on the west, about 180 feet above the sea.

In this basin sandstone and limestone rocks, of the oolitic age make their appearance.

2. *Old (apparently) Sea-Beaches.*

These occur at two spots, one in the S.E. corner of the island, at a height of about 60 feet above the sea,—the other on the N.W. end of the island, about 100 feet above the sea. They consist of pebbles of rolled gravel, mostly half inch to three inches in length closely compacted.

Except where these beaches occur, there are no gravel beds known on the island.*

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* *Note by the Convener.*—In the 6-inch Ordnance Survey map of Rum, two "gravel pits" are indicated as existing in the N.E. part of the island at a height of about 200 feet above the sea.

The soil everywhere consists, apparently, of the débris of volcanic rocks.

But wherever the surface is cut through, as by water courses, small stones of a kind of *granite*, from 6 to 18 inches long, are found.

Most of the walls on the island contain stones of this description gathered off the surface of the land.

Near the Manse there is a wall and also a dam for water, at about 221 feet above the sea, in which there are granite blocks from one to three feet in length.

3. *Boulders.*

1. The largest measured for size rests on the *Scoor* ridge, being 4½ feet long, 4 feet 3 inches broad, and the thickest part 4 feet. It is angular, and may be gneiss. It has a considerable vein of quartz in it.

It is near the western extremity of the ridge, and on a part of the ridge which is lower than any other part, viz., 890 feet above the sea.

It is close to the top of the ridge, on the slope facing the north, and so precariously posed, that the least agitation or concussion by ice, or even water, might be expected to topple it down hundreds of feet.

2. There are other boulders of the same species of granite or gneiss, at heights of from 200 to 700 feet above the sea, on ground sloping from the *Scoor* ridge towards the N.E., and also from the opposite hill down towards a water channel.

In a wall there are numbers of granite blocks of such a size as a strong man can lift.

3. In the N.E. part of the island there is a granite boulder, dark in colour, of a larger size than any other. It is on the part of the hill sloping down towards the S.W., about 600 or 700 feet below the top of the hill, and about 300 feet above the sea. A rock very like that composing this boulder (Mr Macpherson states) he has seen on the shores of Loch Alsh, situated to the east of Skye, about 50 miles north of Eigg.

The boulders (Mr Macpherson states) "are of every variety of gneiss and granite, some very dark, some very white, some red, some with little or no mica, some with a great deal."

Mr Macpherson adds that there are no gneiss or granite rocks in the island; that he has seen rocks of that description in islands and on the mainland, to the N.W., N., and N.E., but that being not sufficiently acquainted with the qualities of these rocks, he cannot venture to indicate the sources of the Eigg boulders.*

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* *Note by the Convener.*—The foregoing report on Eigg having been submitted to Professor Heddle, he wrote to the Convener that he had never visited Eigg, but that he was well acquainted with the rocks in Rum, Mull, and Skye. He states that there is no *true* granite in Rum; but that at the S.W. corner of the island there is a large mass of a "variety of syenite, something like grey granite"; and that this syenite, and also the augite rock of the island, are of the "same type as that of the Coolins in Skye, and of St Kilda." He adds that this augite rock is so peculiar, that if the Eigg boulders came from any of the above-mentioned rocks in these islands, he might perhaps be able to recognise them, on getting chips from the boulders.

In consequence of this last remark by Professor Heddle, the Convener applied to Mr Macpherson, to endeavour to obtain chips of the larger boulders in Eigg. In the course of a few weeks about a dozen chips were obtained, and were sent to the Convener.

The Convener thereupon forwarded them to Professor Heddle, who, after a first cursory examination, wrote to the Convener as follows:—"None of the boulders (judging by the chips) are from any point I know. None are granitic. One is a micaceous syenite, with characteristic radiation structure in its felspar. There is a micaceous gneiss—I think a Tiree rock. The others are either highly metamorphosed grits, simulating granites, or gneissose rocks. All are so characteristic that their source is certain to be sooner or later discovered. The island should be visited, to note carefully the features of the '*lie*' of each boulder."

A few days afterwards Professor Heddle wrote to the Convener, that having again examined the chips he found that "they consist of highly metamorphosed grits of gneiss, and of syenite;—they are all of much greater age than any rocks described as occurring in Eigg"; and suggested that they might be submitted to Mr Archibald Geikie and also to Professor Judd, both of whom he knew were well acquainted with West Highland rocks.

The Convener accordingly transmitted the chips to Professor Judd, with a request that if Mr Geikie was in London, he would have the goodness to show them to him.

The Convener has received a note from Professor Judd stating that Mr Geikie and he had looked at the specimens, and he fixes on one which he says "I take for Torridon Sandstone. It is an Arkose, which Mr Geikie thinks may have come from the Torridon group;—but that it reminds him most of some parts of the Old Red Sandstone. None of the boulders are at all like any of the volcanic masses of the Inner Hebrides."

"All the boulders, we both agree, may have come from the great gneiss masses of the central Highlands, and they do not appear to belong to the old Lewisian or Laurentian series. Mr Geikie adds, 'I do not think anybody could venture to fix their source more precisely'; and in this I quite agree with him

Mr Geikie, in his account of the Geology of Eigg, adverts to the channe

VI.—*NOTES ON FLINT NODULES FOUND ON RAISED SEA-BEACHES AND DRIFT GRAVEL IN WIGTOWNSHIRE. BY REV. GEORGE WILSON, COR. MEM. S.A. SCOT.*

Mr Milne Home asks me (January 1883) for an extract on this subject from an article by me (in vol. i. of the *Collections of the Ayrshire and Wigtownshire Archæological Association*), on the Ancient Stone Implements of Wigtownshire, and to give some additional notes. The passage occurs at page 4.

"The Glenluce implements of flint and other kinds of stone, and of bronze, were first described in 1876, in my notes of some of the articles then presented by me to the Museum of Antiquities in Edinburgh (*Proc. Soc. Antiq. Scot.*, vol. xi. pp. 580–587). The Glenluce flints, &c., are chiefly found on or near certain old sea-beaches at the north shore of the Bay of Luce. These are about 20 feet above the sea-level, and run from north-east to south-west, in parallel storm-beaches, from a point near Park Hay, in Glenluce, to a point near Sandhead, in Stoneykirk, a distance of about six miles. These beaches are in most places covered by sand hills, called the Torrs. They contain many *water-worn nodules of flint*. How did these flints get there? In the paper referred to, I hazarded the opinion that they are 'the relics of a (vanished) *Scottish deposit of chalk*:' but geologists demurred to this, and were inclined to think they had been imported as articles of commerce. One correspondent, who is an eminent geologist, thought they had been brought in *coracles* from the north of Ireland, where flint is plentiful. I am now able to state that *they have been deposited by natural agency*, for I have found them in the *stratified gravel*, in a large excavation at Dunragit railway station, and in a gravel pit at Genoch, which is near some of the old beaches where I have found flints, both wrought and unwrought. It is for geologists to discuss whether they have drifted from the north of Ireland

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of an ancient river, in which he detected "pieces of *red sandstone* 'of Cambrian derivation'—which make it clear that the higher grounds from which they were borne could not have lain to the S. or E., but *to the N. W. or North.*" From fragments of white sandstone (also found by him), he says, "We may with some probability infer, that the course of the stream (which brought them came from the north, where the great white oolite sandstones rise to the surface."—*Lond. Geol. Soc.*, vol. xxvii. p. 309.

or some other quarter. For archæologists it is an interesting question whether this deposit of drift contains chipped flints of the palæolithic period. As yet I have found none."

In a paper in the *Proc. Soc. Antiq. Scot.* for 1881, new series, vol. iii. p. 262, I note that these parallel beaches "are also seen in the cultivated fields in the farm of Culmore in Stoneykirk."

The fact that many of the flints are not as large as a pea, led me to think that they were not imported by human agency. After repeated search, I found flints *in situ* in the strata of drift in the gravel pits at Dunragit station at 70 feet level, Genoch at 25, and at other places. This fact explains their presence on the raised beaches and in the bed of streams.

Near Glenluce three successive sea-beaches are marked by three terraces, more or less distinct. The lowest, from 15 to 25 feet above the present sea-level, is well marked in many parts of Wigtownshire, and contains many caves worn in the Lower Silurian rocks. The two higher lines are carried to the north and west of Castle of Park, by sand hills, now cultivated. The second beach is about 60 or 70, and the third about 100 feet above the present sea-level. In the highest the sand is covered by stratified gravel from 3 to 6 feet deep. A fine section is seen in the cutting of the Girvan Railway on West Borland farm. I have found no flints there as yet; but there are pebbles of a soft red sandstone, some of which are larger than I can lift. Such pebbles were sometimes used as whetstones, and for other purposes, by the ancient stone-workers. (A broken valve of a species of astarte was got about three feet below the surface at this section.)

In Kirkmaiden parish I have found *large rolled flints* in the stratified gravel at the sea-beach near Drumore village, and *smaller* ones in two gravel pits near Logan House, about 50 feet above the sea. I got a single flint in the cutting exposed by the road at the side of the lighthouse buildings at the head of the Mull of Galloway, about 200 feet above the sea. They are also to be found in a gravel pit near Lochnaw Castle, about 175 feet above the sea, and at Machar, in Inch, about 75 feet.

The great mass of the stones in our drift beaches and river beds consists of pebbles of grey Silurian sandstone. There are many specimens of granite, chiefly grey. Boulders and pebbles of red granite are more frequent further west in the Rhinn district. Quartz and quartzite pebbles are not uncommon. Many of those on the sand hills have been used as hammer stones. There are pebbles of various porphyries and other rocks which would repay examination by trained geologists. I believe some of the materials of our drift have come from Arran. Flint is plentiful *in situ* in Armagh. At Glenluce the drift is not found very far from the sea. The sections at higher levels show much boulder clay. As yet I have seen no flint in the boulder clay.

VII.—*NOTES BY REV. PROFESSOR DUNS ON BOULDERS IN THE ISLAND OF MULL.*

The Convener having learnt that Professor Duns, one of the Committee, had resided during a portion of last summer in Mull, wrote to ask him whether he had taken note of any boulders.

The following is extracted from Professor Duns' answer, dated 27th April:—

"I spent two months in Mull last year, limiting my wanderings to the northern part of the island.

"Perhaps the best answer to your kind note is to copy from my diary the only notes I made on boulders.

"June 3, 1882, *Tobermory.*—Walked to *Mishnish* first lake. Struck into the hills at the burn on the north. A boulder of coarse reddish granite, on the pretty steep slope of the east bank. This is the first '*heathen*' I have seen. None were met with on the moor across which I passed yesterday, lying between the new road to Sorn and the Sound of Mull.

"June 7th.—Yesterday's drive from *Tobermory* to *Torloisk*, and to-day's ramble among the hills between the *Sorn* road and the *Sound*, and chiefly those lying above the *Runa-Gal* lighthouse, correct my note under 3rd current. Both on each side of the road to *Torloisk* and among these hills, granite boulders are numerous, but few are large. They range in size from that of a 'fist' to twice that of the 'head.' A few well sunk in the soil show a surface 2 to 3 feet broad. The granite is for the most part a coarse *reddish* one. But there are some fine grained '*greys*.' The largest met with is gneiss. A small *quartzite* specimen occurs in the same area.

All are rounded and smooth. Four lie '*en trainée*' widely apart, the line being N. and S.

"June 22nd.—*Tobermory* to more than 3 miles beyond *Calgary*, that is, about 15 miles. As you approach the west coast of North Mull, along this track, the number of boulders gradually decreases, until, say about *Calgary*, I failed to find any in the parts in which I wandered. The contrast is most striking in this respect, between this track and that between *Tobermory*, south-eastward to *Pennygown*, lying at the seaward opening of *Glen-Forsa*. Has ice, moving from the north-west, begun to drop its entangled boulders near the west coast, and the rate of deposit increase as it passed over the track between *Runa-Gal* and *Mishnish*, between *Mishnish* and the S.E. of *Glen Frisa*, and then through *Glen Aros?* Be this as it may, there is no doubt as to the numerical increase of the boulders in this direction. They are all much worn and rounded, and for the most part comparatively small, though some are large enough to have been utilised as gate-posts.

"July 29.—Ascended *Spyon More*, 2435 feet above sea-level. Wandered in search of boulders, of which there are a good many scattered over the hill and on the heights in the immediate neighbourhood;—all, so far as could be ascertained, *granites*—no granite occurring *in situ* in this part of Mull. Four, widely separated, are lying '*en trainée*.' A line drawn from these across the *Sound of Mull*, and over the *Ardnamurchan* hills, would, if extended, pass between *Eigg* and *Rum*. One of them lies on the *very top* of *Spyon More*;—another is met with half-way down the hill. The others lie in the plain out of which the hill rises. The rocks at the summit are well glaciated, and a great heap of moraine-like débris rests on it. Brought away chips from these boulders."

3. On a new Entozoon (*Pentastomum protelis*) from the Mesentery of *Proteles cristatus*, Sparrmann. By W. E. Hoyle, M.A. (Oxon), F.R.S.E., Naturalist to the "Challenger" Expedition Commission.

(*Abstract.*)

The entozoon, which is a new species of *Pentastomum*, was found by Professor Morrison Watson in a male *Proteles cristatus* which had died in the Zoological Society's Gardens.

The parasites, to the number of about ten, were found enclosed in cysts in the mesentery, and presented the appearance shown in fig. 1. In every instance except one, the ventral surface of the animal formed the convexity of the curve.

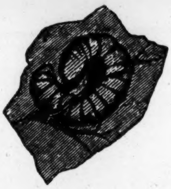
Fig. 1. General appearance of the encysted parasite.

The species may be characterised as follows:—

Pentastomum protelis, n. sp.

Body, cylindrical in the anterior half, slightly tapering posteriorly, terminal segment obtusely pointed. No clear distinction between cephalothorax and abdomen. Head hemispheroidal, equal in diameter to the body. Mouth furnished with a papilla, perhaps a protrusible proboscis. Stigmata arranged in numerous irregular rows on all the segments. Male, 13–17 *mm. in length, with* 16 *or* 17 *annuli. Female,* 20–25 *mm. in length, with* 18–22 *annuli. Habitat, the mesentery of* Proteles cristatus, *enclosed in a connective tissue cyst.*

The above description being taken from specimens which, from their encysted condition, were presumably immature, must be regarded, to a certain extent, as provisional.

The appearance of the ventral surface of the head is shown in fig. 2. In one or two examples the posterior terminal segment aptly fulfilled the expression used by Diesing in describing another species, "cute externâ in formâ præputii," but this was by no means constant.

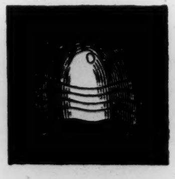
Fig. 2. Ventral surface of the head.

The species bears a close resemblance to *P. polyzonum*, Harley, but is distinguished from it by the number of segments, which, in the case of *P. polyzonum*, is stated to be constantly nineteen in fully developed females,* and it is very unlikely that an immature form should have more, though it might have fewer, segments than the adult.

With respect to the anatomy of the animal, the following points resulting from this investigation seem to be most worthy of notice.

The *cuticle* is perforated by the usual stigmata, which are seen in section to be somewhat of an hour-glass shape, and immediately internal to each is situated a spheroidal cell or group of very small cells (fig. 3, *d*), with a blunt process projecting into the stigma, the arrangement very forcibly suggesting that the function of this aperture is to evacuate a substance secreted by the cells.

The sub-cuticular epidermis (fig. 3, *b*) consists of columnar nucleated cells, and in many cases processes can be traced passing from these inwards and becoming connected either with the muscle fibres or with the cells forming the parenchyma of the body wall.

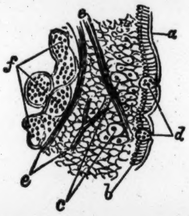

Fig. 3. Transverse section of a portion of the body wall: *a*, cuticle; *b*, layer of epithelium; *c*, large glandular cells; *d*, large cells just within the stigmata; *e*, transverse muscular fibres; *f*, longitudinal fibres.

The *mouth* is annular in form and surrounds a small oval papilla, which is probably capable of protrusion and retraction, seeing that it it is provided with a sphincter muscle, with other muscles placed longitudinally with respect to the body, and with a third group passing inwards at right angles to these.

The *œsophagus* commences at the posterior side of this papilla, and passes inwards and backwards with a double curve, finally entering the stomach some distance behind its anterior margin. At first its section is crescentic, with the horns directed dorsally. It then becomes oval, and still later crescentic again, but with the horns directed ventrally. All this portion of the tube is lined with chitin continued from the external covering of the body. The middle portion of the œsophagus is once more oval in section, and has a well-developed coat of muscular fibres. On passing through the nerve-ring its lumen becomes stellate in section, and it enters a mass of rather large round parenchymatous cells, whose appearance suggests that they are secretory in function, though as yet no excretory passage has been noticed. Possibly they lead by very minute openings into the œsophagus itself, and perform the function of a salivary gland.

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* Bell, *Ann. Mag. Nat. Hist.*, ser. 5, vol. vi. p. 176.

The *intestine* or *stomach*, for the alimentary canal presents no division into these two parts, has a straight antero-posterior course, and terminates by a narrow rectum at the hinder extremity of the body. Two mesenteries, not diametrically opposite to each other, but separated by an angle of 90° to 120°, pass from the body wall to the intestine, enclosing also two large glandular bodies (" Hakendrüsen " of Leuckart) which lie one on either side of it.

The *testis* is a long thin-walled dorsally-situated sac, as described by previous observers. Towards its anterior extremity the lower wall presents a thickening in which two grooves are formed, whose walls arch over, and convert them into two tubes, the vasa deferentia, which subsequently become united, and then separate again.

Their course is now round the intestine, one on each side, passing through the " Hakendrüsen," until they reach the ventral aspect of the body, where each becomes connected with the excretory apparatus of its own side.

The *genital apertures* lie side by side a short distance behind the mouth, and each leads into an oval sac, to all appearance the homologue of that which contains the cirrus in *P. tænioides*, except that in the specimens examined it was quite empty.

From the dorso-lateral side of this sac, on which aspect its wall is very much thickened, two tubes are given off, one anterior and one posterior. The anterior tube enlarges, soon after its commencement, to form a small spheroidal chamber, and, subsequently contracting, passes backwards, extends some distance behind the sac, and terminates blindly after a course of three or four millimetres; the end of it is variously coiled.

A little anterior to the middle of this tube the vas deferens comes into contact with it, the walls of the two fusing, but the lumen of the vas deferens is not continued beyond this point; *it terminates blindly.*

The truth of this observation, so exactly opposed to what might have been expected, has been confirmed by the preparation of a very careful series of transverse sections, which show the one tube lined with a perfectly smooth layer of epithelium, passing by the other, utterly ignoring, if the expression may be allowed, its proximity, while the vas deferens fades away in a mass of rather loose cells without showing the least desire to unite with its companion.

According to Leuckart's account of the development of these organs in *P. tænioides*, there is first formed a continuous tube extending from the testis to the external surface, and the curved cæcal tube is formed later as an outgrowth from it;* this makes it more difficult to understand why a stoppage should occur in a tube which was once open, and which must be open again in the adult condition of the animal, in order to allow of the functional activity of the reproductive organs.

The posterior of the two tubes given off from the sac above mentioned does not extend backwards beyond the sac itself; its lumen is almost filled, and reduced to a crescentic form by a cylindrical body, attached by one side to the interior of the tube; anteriorly it terminates in a sharp point, and is clearly homologous with two conical bodies which Leuckart has described in *P. tænioides* under the name "Chitinzapfen," † and which he regards as being in some way accessory to the process of reproduction.

BUSINESS.

The following Candidates were balloted for and declared duly elected Fellows of the Society:—Mr J. B. Readman; Mr Henry Newcombe, F.R.C.S.E.; Mr Charles Watson.

Monday, 18th June.

PROFESSOR DOUGLAS MACLAGAN, Vice-President, in the Chair.

The Chairman read a Memoir of the late Sir Wyville Thomson, drawn up by Professor Redfern.

The following Communications were read:—

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* Leuckart, *Bau u. Entwickelungsgesch. d. Pentastomen*, 1860; S. 134, tab. iv. fig. .

† *Loc. cit.*, S. 78.

1. Bright Clouds on a Dark Night Sky. By the Astronomer-Royal for Scotland.

2. Mathematical Note. By Mr A H. Anglin.

3. Note on the Compressibility of Water, Sea-Water, and Alcohol, at High Pressures. By Professor Tait.

The apparatus employed was of a very simple character, similar to that which was used last autumn in the "Triton."

It consisted of a narrow and a wide glass tube, forming as it were the stem and bulb of a large air thermometer. The stem was made of the most uniform tube which could be procured, and was very accurately gauged; and the weight of the content of the bulb in mercury was determined. Thus the fraction of the whole content, corresponding to that of one millimetre of the tube, was found.

This apparatus had the interior of the narrow tube very carefully silvered; and while the whole, filled with the liquid to be examined, was at the temperature of the water in the compression apparatus, the open end was inserted into a small vessel containing clean mercury. Four instruments of this kind were used, all made of the same kind of glass.

The following are the calculated apparent average changes of volume per ton weight of pressure per square inch (*i.e.*, about 150 atmospheres):—

FRESH WATER, at 12° C.

Pressure.	1	2	3	4	Mean.
1	0·00670	*	665	666	0·00667
2	0·00657	*	646	656	0·00653
2·5	0·00651	650	640	648	0·00647
3	0·00641	633	636	636	0·00636

Note.—The first two experiments with No. 2 failed in consequence of a defect in the silvering.

The compressibility of the glass was not directly determined. It may be taken as approximately 0·000386 per ton weight per square inch.

From these data, which are fairly consistent with one another, we find the following value of the *true* compressibility of water per ton, the unit for pressure (p) being 1 ton-weight per square inch, and the temperature 12° C.,

$$0{\cdot}0072\,(1-0{\cdot}034\,p)\,;$$

showing a steady falling off from Hooke's law.

SEA-WATER, at 12° C.

Pressure.	1	2	3	4	Mean.
1	0·00606	611	615	627	0·00615
2	0·00595	607	598	601	0·00600
2·5	0·00600	600	594	590	0·00594
3	0·00588	593	586	586	0·00588

Note.—The sea-water employed was collected about 1½ miles off the coast at Portobello.

These give, with the same correction for glass as before, the expression

$$0{\cdot}00666\,(1-0{\cdot}034\,p).$$

Hence the relative compressibilities of sea and fresh water are about

$$0{\cdot}925:$$

while the rate of diminution by increase of pressure is sensibly the same (3½ per cent. per ton weight per square inch) for both.

With the same apparatus I examined alcohol, of sp. gr. 0·83 at 20° C.

ALCOHOL, at 12° C.

Pressure.	1	2	3	4	Mean.
1	0·01202	1193	*	*	0·01200
2·5	0·01040	1052	1050	1056	0·01049
3	0·01043	1050	1043	1058	0·01048

These experiments were not so satisfactory as those with water. There are peculiar difficulties with the silver film. I therefore make no definite conclusion till I have an opportunity of repeating them.

I intend to perform the whole of these experiments at other temperatures, with the identical apparatus, as soon as possible next winter.

Monday, 2nd July 1883.

Mr ROBERT GRAY, Vice-President, in the Chair.

The following Communications were read:—

1. Note on the little *b* group of lines in the Solar Spectrum, and the new College Spectroscope. By the Astronomer-Royal for Scotland.

2. On Superposed Magnetisms in Iron and Nickel. By Professor C. G. Knott, D.Sc.

(*Abstract.*)

When an iron wire is magnetised longitudinally, it lengthens in the direction of magnetisation, according to an old discovery of Joule's. More recently Wiedemann showed that when wire is at the same time magnetised circularly it tends to twist. Thus, if an iron wire be fixed at one end, and stretched vertically by means of a mass attached to the free end, the free end will twist round when the wire is both traversed by one current and magnetised by another which traverses a helix surrounding it. If the wire is magnetised so as to have the north pole down, a down current will make the free end twist in the direction of the hands of a watch as looked at from above. Reversal of either current reverses the direction of twist; reversal of both produces no alteration. Maxwell and Chrystal have pointed out that Wiedemann's phenomenon can be explained by means of Joule's.

In the experiences that form the subject of the paper this was verified, and other peculiarities in the twisting of iron under the influence of these magnetisations were discovered which tend more securely to establish this relation. Thus, when the linear current (the current along the wire) was kept constant, the twist reached a maximum for an intermediate value of the helical current, this critical value being greater for a greater linear current—a result in remarkable harmony with the details of Joule's discovery.

Again, for different tensions, the twisting under the influence of two given magnetisations varied, being (with one or two exceptions)

greater for the smaller tension—precisely as Joule's results would lead one to expect. This effect of tension seems to have escaped the notice of Wiedemann, but it was undoubted.

In the case of nickel, the direction of the twist was different from that for iron, a conclusion which Barrett's discovery of the *contraction* of nickel under magnetisation rendered not unexpected. No maximum twist was reached for an intermediate current. Otherwise the results were the same as for iron, as, for example, in the case of the tensions.

The experiments were tried with different thicknesses of wire, and all gave the same conclusions.

3. Further Note on the Maximum Density Point of Water.

By Professor Tait.

During my long investigations of the "Pressure-Errors of the 'Challenger' Thermometers," but more especially two years ago (*Proceedings*, May 1881), I was led to suspect a lowering of the maximum density point of water by pressure. For I found that the change of temperature of water increased *faster* than in direct proportion to the sudden change of pressure which produced it. These experiments were, at my request, more fully carried out by Messrs Marshall, Smith, and Omond (*Proceedings*, July 1882). Their result was (approximately) a lowering of the maximum density point by 5° C. for 1 ton-weight of pressure per square inch (roughly speaking, about 150 atmospheres). In a note appended to their paper I deduced from their experimental data a lowering of 3°·6 C., and from my own a lowering of about 3° C. for the same pressure.

In November last, when reducing the observations made in the "Triton" (*ante*, p. 45), I found that water becomes more compressible as its temperature is lowered, at least down to 3° C.; and this I regarded as another indirect proof of the lowering of the maximum density point by pressure.

I then determined to try a direct process analogous to that of Hope, for the purpose of ascertaining the maximum density point at different pressures. The experiments presented great difficulties, because (for Hope's method) the vessel containing the water must have a considerable cross section; and thus I could not use my smaller compression apparatus, which was constructed expressly to admit of measurements of temperature by thermo-electric processes. I had therefore to work with the huge Fraser gun employed for the "Challenger" work, and to use the protected thermometers (which are very sluggish) for the measurement of temperatures. It was also necessary to work with the gun at the temperature of the air,—it would be almost impossible to keep it steadily at a much lower temperature,—so that I had to work in water at about 12° C.

The process employed was very simple. A tall cylindrical jar full of water had two "Challenger" thermometers (stripped of their vulcanite mounting) at the bottom, and was more than half-filled with fragments of table-ice floating on the water, and confined by wire-gauze at the top. This was lowered into the water of the gun, and pressure was applied.

It is evident that *if there were no conduction of heat* through the walls of the cylinder, and if the ice lasted long enough under the steadily maintained pressure, the thermometers would ultimately show, by their recording minimum indices, the maximum density point corresponding to the pressure employed :—always provided that that temperature is not lower than the melting point of ice at the given pressure.

Unfortunately, all the more suitable bad conductors of heat are either bodies like wood (which is crushed out of shape at once under the pressures employed) or like tallow, &c. (which become notably raised in temperature by compression). I was therefore obliged to use glass. The experiments were made on successive days, three each day, with three different cylindrical jars. These had all the same height and the same internal diameter. The first was of tinned iron; the second of glass about $\frac{1}{6}$ inch thick; the third, of glass nearly an inch thick, was procured specially for this work.

With the external temperature 12°·2 C., the following were the results of $1\frac{1}{2}$ tons pressure per square inch, continued in each case for 20 minutes (some unmelted ice remaining on each occasion). The indications are those of two different "Challenger" thermometers, corrected for index-error by direct comparison with a Kew standard :—

Tin Cylinder.	Thin Glass.	Thick Glass.
4° C.	2°·67	0°·83
4°	2°·61	0°·83

The coincidence of the first numbers with the ordinary maximum density point of water is, of course, mere chance. When no pressure was applied, but everything else was the same, the result was

Tin.	Thin.	Thick.
5°·7 C.	5°	4°

It is clear that the former set of numbers points to a temperature of maximum density, somewhere about 0° C., under 1½ tons pressure per square inch. But still the mode of working is very imperfect.

I then thought of trying a *double* cylindrical jar, the thin one above-mentioned being enclosed in a larger one which surrounded it all round, and below, at a distance of about ¾ inch. Both vessels were filled with water, with broken ice floating on it, and had "Challenger" thermometers at the bottom. By this arrangement I hoped to get over the difficulty due to the temperature of the gun, by having the inner vessel enclosed in water which would be lowered in temperature to about 3° C. by the application of pressure. The device proved quite successful. The result of 1½ tons pressure per square inch maintained for 20 minutes, some ice being still left in each vessel, was from a number of closely concordant trials—

Temperature in outer vessel, . .	1°·7 C.
Temperature in inner vessel, . .	0°·3 C.

The direct pressure correction for the thermometers is only about $-0°\cdot1$ C., and has therefore been neglected.

The close agreement of this result with that obtained (under similar pressure conditions) in the thick glass vessel leaves no doubt that the lowering of the maximum density point is somewhat under 4° C. for 1½ tons, or 2°·7 C. for 1 ton, per square inch. It is curious how closely this agrees with the result of my indirect experiments.

The fact, that an increasing compressibility of water at lower temperatures points to a lowering of the maximum density point by pressure, is easily seen from the consideration of the *surface* which represents the density of water in terms of pressure and temperature as independent variables. Let temperatures in degrees C. be measured on x, pressures in atmospheres on y, and density on z. Then the section by the plane $y=1$ is a curve with a maximum at $x=4$. But from this section the surface rises, in the direction of y positive, faster for lower values of x (*i.e.*, $\frac{dz}{dy}$ for $y=1$ becomes greater as x diminishes). Hence it is clear that a proximate section, say for $y=2$, has its maximum ordinate for a value of x *less* than 4° C.

In fact, if e be the expansibility of water, it can be expressed as a function of the temperature and pressure alone, *i.e.*,

$$e=f(t,p).$$

Hence simultaneous changes of temperature and pressure, which leave the expansibility unaltered, are connected by the relation

$$\left(\frac{de}{dt}\right)\delta t+\left(\frac{de}{dp}\right)\delta p=0.$$

The maximum density point is a particular case of this, for there the expansibility is zero.

Now, if v be the volume, we have

$$e=\frac{d}{dt}(\log v),$$

so that

$$\left(\frac{de}{dp}\right)=\frac{d^2}{dpdt}(\log v)=\frac{d}{dt}\left(\frac{d}{dp}(\log v)\right).$$

But $-\frac{d}{dp}(\log v)$ is the compressibility, and diminishes with increase of t. Thus $\frac{de}{dp}$ is essentially positive. But so is $\frac{de}{dt}$ so long as the temperature is above the maximum density point. Thus δp and δt in the above equation *have opposite signs*.

I have learned quite recently, and by mere accident, that the lowering of the maximum density point of water has been already pointed out as a theoretical result by processes essentially the same as that just given; first by Puschl[*] in 1875, then by Van der Waals[†] in 1876. Both authors refer to experiments made by Grassi[‡] for Regnault in 1848, on the compressibility of water at different temperatures. Grassi's results were all obtained for pressures of some 10 atmospheres or so at the utmost; and, if they may be trusted, indicate a most extraordinary relation between the compressibility and the temperature of water within a few degrees of 4° C. It seems almost hopeless to attempt to obtain an approximate value of $\frac{de}{dp}$ at 4° C. from Grassi's table. Yet this has been attempted by both of the authors above named.

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[*] *Kais. Ac. d. Wiss. Sitzb.*, lxxii. 283.

[†] *Archives Néerl.*, xii. 457.

[‡] *Annales de Chimie et de Physique*, ser. iii. t. 31, 1851.

Puschl calculates that 87·6 atmospheres lower the maximum density point by 1° C. Thus he (virtually) gives the effect of 1 ton per square inch as 1°·7 C. instead of 2°·7 C. as above. Van der Waals gives a much more rapid descent, *i.e.*, from 4°·08 C. at 1 atmosphere to 3°·4 C. at 10·5 atmospheres. He also gives the results of some experiments with the ordinary piezometer, and therefore at very moderate pressures, which indicate a lowering of the maximum density point, but do not lead to definite numerical results.

In order to clear up the whole subject, I intend to repeat all these experiments next winter, when the gun and its contents are naturally at a temperature of about 4° C.

4. On Surface Emissivity. By Professor Tait.

BUSINESS.

Professor Tait laid before the Society a photograph of the arm of a child struck by lightning, sent by direction of Mr Milne Home.

Dr J. Graham was balloted for and declared a duly elected Fellow of the Society.

Monday, 16*th July* 1883.

THE RIGHT HON. LORD MONCREIFF, President,
in the Chair.

The Chairman read a Communication from the Science and Art Department; and an Obituary Notice of the late Mr T. W. Rumble.

The following Communications were read:—

1. On a proposed Edinburgh Marine Station for Biological Research, at Granton Quarry. By Mr John Murray.

2. On Work done on board H.M.S. "Triton" in the Faroe Channel during the Summer of 1882. By Mr John Murray.

3 On the *Pennatulida* dredged in the Faroe Channel during the Cruise of H.M.S. "Triton" in August 1882. By Professor A. M. Marshall. Communicated by Mr John Murray.

4. On the *Asteroidea* dredged in the Faroe Channel during the Cruise of H.M.S. "Triton" in August 1882. By Mr W. Percy Sladen, F.L.S., F.G.S. Communicated by Mr John Murray.

5. On the *Pycnogonida* dredged in the Faroe Channel during the Cruise of H.M.S. "Triton" in August 1882. By Dr P. P. C. Hoek. Communicated by Mr John Murray.

6. On the *Crustacea* dredged in the Faroe Channel during the Cruise of H.M.S. "Triton" in August 1882, By Rev. A. M. Norman, D.C.L. Communicated by Mr John Murray.

7. On the *Tunicata* dredged in the Faroe Channel during the Cruise of H.M.S. "Triton" in August 1882. By Professor W. A. Herdman.

8. On the Proofs of Proportionality of Emissive and Absorptive Power. By Professor Tait.

9. A Contribution to the Chemistry of Nitroglycerine. By Matthew Hay, M.D., Assistant to the Professor of Materia Medica in the University of Edinburgh. Communicated by Professor Crum Brown.

(A full report of this paper appears in the *Transactions* of the Society.)

On account of the identity of the physiological action of nitroglycerine with that of simple alkaline nitrites, it occurred to the author either that nitroglycerine was not a nitrate of glyceryl as commonly represented, but contained nitrous acid in its constitution, or that it was readily decomposed in the body with the formation of nitrous acid. This suggestion led to an extended investigation of the constitution of nitroglycerine and of the decomposing action of various substances on nitroglycerine. The conclusion arrived at as to the constitution of nitroglycerine was confirmatory of the common belief that it is a tri-nitrate of glyceryl. For it was found that although nitroglycerine yielded a large quantity of nitrous acid when decomposed by alkalies, yet nitroglycerine could not be obtained by acting on glycerine with nitrous anhydride gas, nor did the presence of urea prevent its formation when it was prepared in the usual way by allowing nitric and sulphuric acids to act on glycerine. The yield of nitroglycerine from a given weight of glycerine also closely agreed with its being a tri-nitrate, and was considerably in excess of the yield obtainable had nitrous acid entered into its constitution. An analysis of the elementary composition of nitroglycerine, which was carried out in conjunction with Mr Masson,* also led to the same opinion.

With respect to the decomposing action of various substances on nitroglycerine, the novel and interesting result was obtained that potash, soda, ammonia, or an alkaline carbonate decomposed nitroglycerine, when heated with it for a longer or shorter period, forming an amount of nitrous acid corresponding almost precisely to the reduction of two-thirds of the nitric acid which nitroglycerine contains. It was also found that the proportion of alkali necessary for this decomposition was 5 molecules of the alkali to 3 molecules of nitroglycerine, and, therefore, a much larger proportion of alkali than has hitherto been believed to be necessary. Further, in direct contradiction of all previous statements, no glycerine could be discovered amongst the products of decomposition, which, besides nitrite of the alkali, consisted of nitrate, acetate, formate, and oxalate of the alkali, a reddish aldehydic resin, and a body capable of gelatinising in alcohol. The amount of nitrous acid was not affected whether alcohol or water was used as the medium in which the alkali was allowed to act on the nitroglycerine.

A solution of phosphate of soda (Na_2HPO4) acted on nitroglycerine in almost the same manner as an alkali, only much less powerfully. Chloride of sodium had extremely little action. Strong solutions of mineral acids, as hydrochloric and sulphuric acids, acted much more slowly than alkalies, and, as was to be expected, only a trace of nitrous acid was found amongst the products of decomposition. Contrary to the experience of De Vrij, sulphuretted hydrogen was found to have no action on nitroglycerine when dissolved either in ether or in alcohol. Alkaline sulphides, on the other hand, acted very energetically on nitroglycerine, and decomposed it even more powerfully than alkalies. The stronger action of the sulphides was especially noticeable when a watery and not an alcoholic solution was allowed to act on the nitroglycerine. Under this condition an alkali decomposes nitroglycerine very slowly, and only after heating for several hours; an alkaline sulphide decomposes it in almost as many minutes.

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* *Vide* following paper.

Nitroglycerine heated for one hour with alcohol alone did not exhibit any signs of decomposition; heated with water for three hours it decomposed slowly.

Some experiments were made to ascertain the highest possible practical yield of nitroglycerine from a given quantity of glycerine, and the best proportion of acids to obtain this yield. The highest yield was over 95 per cent. of the theoretical yield, and this was obtained by adding 1 part of glycerine to 9 parts of an acid mixture, consisting of 2 parts of strong sulphuric acid to 1 part of fuming nitric acid. The yield was not improved by substituting fuming sulphuric acid for the ordinary strong acid; and the yield was greatly lessened when ordinary strong nitric acid was substituted for fuming nitric acid. The yield was not to any considerable extent increased by allowing the acids to remain in contact with the glycerine for a longer period than five minutes. By whatever method, or with whatever proportion of acids the nitroglycerine was prepared, the products were all found to be uniform in their physical and chemical characters. Nitroglycerine does not therefore appear to be a body of variable composition as some chemists assert (Hess). It is always perfectly colourless, and not yellowish as is sometimes stated. The yellow colour is probably for the most part due to the nitroglycerine having been washed in the process of its manufacture with a solution of soda, which decomposes it.

Its solubilities were carefully ascertained, and it was found to be very slightly soluble in water; largely soluble in ethylic alcohol, but much more soluble in methylic alcohol or methylated spirit; and soluble in every proportion in ether, chloroform, glacial acetic acid, and carbolic acid; freely soluble in benzol; difficultly soluble in carbon disulphide; and practically insoluble in glycerine.

On account of the constancy observed in the amount of nitrous acid produced when nitroglycerine is decomposed by an alkali, the author suggests a method for estimating nitroglycerine by decomposing it with potash or soda in alcoholic solution, and afterwards determining the amount of nitrous acid by means of starch and an iodide.

10. The Elementary Composition of Nitroglycerine. By Matthew Hay, M.D., and Orme Masson, M.A., B.Sc. Communicated by Professor Crum Brown.

(A full report of this paper appears in the *Transactions* of the Society.)

Investigations have been made by Railton, Williamson, Hess and Schwab, Beckerhinn, and Sauer and Ador for the purpose of ascertaining the elementary composition and the constitution of nitroglycerine. They all agree in regarding it as a nitrate of glyceryl; but, whilst some consider that it is a tri-nitrate, others hold that it is a variable mixture of the tri-nitrate with di-nitrate and mono-nitrate. Their analyses are quite insufficient to establish either the one or the other conclusion, and have mainly been confined to estimations of the nitrogen. If we except a comparative estimation of the carbon with the nitrogen, there exist absolutely no determinations of the carbon or of the hydrogen. And, as the decomposition of nitroglycerine with potash has been shown to occur in a manner considerably different from that suggested by Railton and Williamson, the main reason in support of the constitution of nitroglycerine as a tri-nitrate has been removed.* The authors of the present communication therefore believed that they were amply justified in making a fresh and more careful and complete analysis of the composition of nitroglycerine. Absolute determinations were made, not only of the nitrogen, but also of the carbon and hydrogen; and, in order to ascertain the uniformity in composition of nitroglycerine, the nitrogen of samples prepared by various methods was estimated. The nitroglycerine was both pure and thoroughly dried. For the determination of the nitrogen, modifications of Dumas's method and of Schloesing's method were employed. The carbon and hydrogen were estimated by a modification of Liebig's method. Every precaution was taken to insure that the results obtained should be correct. The average of the determinations gave 15·91 per cent. of carbon, 2·49 per cent. of hydrogen, and 18·05 per cent. (Dumas) or 18·14 per cent. (Schloesing) of nitrogen. Theoretically nitroglycerine, regarded as the tri-nitrate of glyceryl, contains 15·86 per cent. of carbon, 2·20 per cent. of hydrogen, and 18·50 per cent. of nitrogen. The quantities obtained by experiment agree so closely with the theoretical quantities that they may be regarded as affording proof that nitroglycerine is, in reality, the tri-nitrate of glyceryl. The authors also conclude, from the unvarying amount of nitrogen obtainable from variously prepared specimens of nitroglycerine, including one from Nobel's dynamite, that nitroglycerine is constant in composition and does not contain any of the lower nitrates of glyceryl, unless very imperfectly washed.

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* *Vide* preceding paper by Matthew Hay.

BUSINESS.

The President, at the close of the meeting, gave a brief review of the Session. He said there had been four Papers on Astronomy, two on Botany, seven on Chemistry, two on Geology, four on Mathematical subjects, six on Meteorology, twenty-three on Natural Philosophy, nine on Natural History, three on Philology, two on Political Economy, and one on Physiology—sixty-three papers in all, independently of the Obituary Notices which had been read. He thought he was not wrong in saying these papers had combined an amount of interest, ability, and novelty which could hardly be excelled, and to a very large extent showing great progress in the subjects treated. He congratulated the Society on having reached the point at which they now stood that night. The Session, he believed, had been a very successful and a very interesting one, and the topics treated of had been of the greatest possible importance. He said, in opening the Session, that it was the 100th Session of the Royal Society, and he thought he used a certain amount of artifice or subterfuge when he made that representation. They had now concluded the 99th Session, and not the 100th. Like other centenarians, their claims up to this point had not been well-founded, as the first year of the century began in November 1783. At that time the Society was divided into two Classes—the Physical and the Literary. The Physical met first on the 4th of November 1783, and the Literary on the 17th of the same month. They would, however, celebrate the commencement of their 100th Session on the first Monday in December. Referring to the division which originally existed in the Society as to Physical and Literary classes, the President said that some one had suggested that they should follow in the footsteps of their predecessors, and have at intervals purely literary papers read in the Society. The reason why that for a Society of that kind scientific researches were more appropriate was that the literary was fluctuating and stationary, whereas the scientific papers read at their meetings marked the progress of science. Still, he had an ambition in some degree or other to resuscitate that good example of their forefathers, and accordingly next Session he would invite any of the Fellows of the Society who felt inclined to form a Literary class. In conclusion, the President said that at the commencement of next Session he would review the hundred years' history of their Society.

The following Paper was read June 18th:—

Mathematical Note. By A. H. Hallam Anglin, M.A., LL.B., M.R.I.A.

If

$$x^m = p_1x^{m-1} + p_2x^{m-2} + p_3x^{m-} + \ldots + p_m \quad . \quad \text{(A.)}$$

then will

$$x^n(n>m) = \mathrm{P}_1x^{m-1} + \mathrm{P}_2x^{m-2} + \mathrm{P}_3x^{m-3} + \ldots + \mathrm{P}_m,$$

where

$$\mathrm{P}_r \equiv p_r \,.\, h_{n-m} + p_{r+1} \,.\, h_{n-m-1} + p_{r+2} \,.\, h_{n-m-2} + \ldots + p_m \,.\, h_{n-2m+r},$$

h_n *being the sum of the homogeneous products of the roots of (A) of n dimensions.*

1. In order to establish this proposition it will be necessary to premise the following lemma :—

If $a_1, a_2, a_3, \ldots a_m$ be roots of the equation

$$x^m = p_1 x^{m-1} + p x_2^{m-2} + p_3 x^{m-3} + \ldots + p_m \, ,$$

then

$$h_n (n > m) = p_1 \, . \, h_{n-1} + p_2 \, . \, h_{n-2} + p_3 \, . \, h_{n-3} + \ldots + p_m \, . \, h_{n-m} \, ,$$

where h_n is the sum of the homogeneous products of $a_1, a_2, a_3, \ldots a_m$ of n dimensions.

The proof of this may be briefly indicated thus :—We have

$$x^m - p_1 x^{m-1} - p_2 x^{m-2} - p_3 x^{m-3} - \ldots - p_m$$
$$= (x - a_1)(x - a_2)(x - a_3) \ldots (x - a_m) \, .$$

Writing $\dfrac{1}{x}$ for x and multiplying off by x^m we get

$$\frac{1}{1 - p_1 x - p_2 x^2 - p_3 x^3 - \ldots - p_m x^m}$$
$$= \frac{1}{1 - a_1 x} \cdot \frac{1}{1 - a_2 x} \cdot \frac{1}{1 - a_3 x} \cdots \frac{1}{1 - a_m x} \, .$$

Thus

$$(1 + \ldots + h_{n-m} \, . \, x^{n-m} + \ldots + h_{n-3} \, . \, x^{n-3} + h_{n-2} \, . \, x^{n-2} + h_{n\cdot 1} \, . \, x^{n-1}$$
$$+ h_n \, . \, x^n + \ldots)(1 - p_1 x - p_2 x^2 - p_3 x^3 - \ldots - p_m \, . \, x^m) = 1 \, ;$$

hence, since this is an identical equation, equating to zero the coefficient of x^n in the first member, we get

$$h_n - p_1 \, . \, h_{n-1} - p_2 \, . \, h_{n \;\, 2} - p_3 \, . \, h_{n-3} - \ldots - p_m \, . \, h_{n-m} = 0 \, ,$$

or,

$$h_n = p_1 \, . \, h_{n-1} + p_2 \, . \, h_{n-2} + p_3 \, . \, h_{n-3} + \ldots + p_m \, . \, h_{n-m} \, .$$

2. We can now establish our proposition—to express $x^n (n > m)$ in the manner indicated above.

For greater clearness we shall first consider a particular case—

$$x^4 = p x^3 + q x^2 + r x + s \, .$$

Multiplying both sides of this equation by x, writing for x^4 its equivalent value, and arranging, we get

$$x^5 = (p^2 + q)x^3 + (pq + r)x^2 + (pr + s)x + ps\,;$$

but by the lemma

$$h_n = ph_{n-1} + qh_{n-2} + rh_{n-3} + sh_{n-4}\,,$$

and in particular $h_1 = p$, $h_2 = ph_1 + q$, $h_3 = ph_2 + qh_1 + r$, &c., where h_n is the sum of the homogeneous products of the roots of $x^4 = px^3 + qx^2 + rx + s$ of n dimensions. Thus

$$x^5 = h_2 \,.\, x^3 + (qh_1 + r)x^2 + (rh_1 + s)x + sh_1\,.$$

Repeating the above process on this equation we shall find, on again applying the lemma, that

$$x^6 = h_3 \,.\, x^3 + (qh_2 + rh_1 + s)x_2 + (rh_2 + sh_1)x + sh_2\,.$$

Now assume

$$x^n = h_{n-3} \,.\, x^3 + (qh_{n-4} + rh_{n-5} + sh_{n-6}) \,.\, x^2 \\ + (rh_{n-4} + sh_{n-5})x + sh_{n-4}\,;$$

operating on this equation in like manner we shall get, since by the lemma $ph_{n-3} + qh_{n-4} + rh_{n-5} + sh_{n-6} = h_{n-2}$,

$$x^{n+1} = h_{n-2}x^3 + (qh_{n-3} + rh_{n-4} + sh_{n-5})x^2 \\ + (rh_{n-3} + sh_{n-4})x + sh_{n-3}\,,$$

which proves the preposition in a particular case.

The general case is established in a similar way,

$$x^m = p_1x^{m-1} + p_2x^{m-2} + p_3x^{m-3} + \ldots + p_m\,.$$

Multiplying both sides of this equation by x, writing for x^m its equivalent value, and arranging the terms, we shall get, since by the lemma $p_1h_1 + p_2 = h_2$,

$$x^{m+1} = h_2 \,.\, x^{m-1} + (p_2h_1 + p_3)x^{m-2} + (p_3h_1 + p_4)x^{m-3} \\ + \ldots + (p_{m-1} \,.\, h_1 + p_m) \,.\, x + p_m \,.\, h_1\,.$$

Applying the same process to this equation, it may be shown, since $p_1h_2 + p_2h_1 + p_3 = h_3$, that

$$x^{m+2} = h_3 \,.\, x^{m-1} + (p_2h_2 + p_3h_1 + p_4)x^{m-2} + (p_3h_2 + p_4h_1 + p_5)x^{m-3} \\ + \ldots + (p_{m-1} \,.\, h_2 + p_m \,.\, h_1)x + p_m \,.\, h_2\,.$$

Now assume

$$
\begin{aligned}
x^n = {} & h_{n-m+1} \cdot x^{m-1} \\
& + (p_2 \cdot h_{n-m} + p_3 \cdot h_{n-m-1} + p_4 \cdot h_{n-m-2} + \ldots + p_m \cdot h_{n-2m+2}) \cdot x^{m-2} \\
& + (p_3 \cdot h_{n-m} + p_4 \cdot h_{n-m-1} + p_5 \cdot h_{n-m-2} + \ldots + p_m \cdot h_{n-2m+3}) \cdot x^{m-3} \\
& \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& + (p_r \cdot h_{n-m} + p_{r+1} \cdot h_{n-m-1} + p_{r+2} \cdot h_{n-m-2} + \ldots + p_m \cdot h_{n-2m+r}) \cdot x^{m-r} \\
& \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& + (p_{m-1} \cdot h_{n-m} + p_m \cdot h_{n-m-1}) \cdot x \\
& + p_m \cdot h_{n-m} ;
\end{aligned}
$$

operating on this equation in like manner it may be finally shown, since by the lemma

$$p_1 \cdot h_{n-m+1} + p_2 \cdot h_{n-m} + p_3 \cdot h_{n-m-1} + \ldots + p_m \cdot h_{n-2m+2} = h_{n-m+2}$$

that

$$
\begin{aligned}
x^{n+1} = {} & h_{n-m+2} \cdot x^{m-1} \\
& + (p_2 \cdot h_{n-m+1} + p_3 \cdot h_{n-m} + p_4 \cdot h_{n-m-1} + \ldots + p_m \cdot h_{n-2m+3}) \cdot x^{m-2} \\
& + (p_3 \cdot h_{n-m+1} + p_4 \cdot h_{n-m} + p_5 \cdot h_{n-m-1} + \ldots + p_m \cdot h_{n-2m+4}) \cdot x^{m-3} \\
& \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& + (p_r \cdot h_{n-m+1} + p_{r+1} \cdot h_{n-m} + p_{r+2} \cdot h_{n-m-1} + \ldots + p_m \cdot h_{n-2m+r+1}) \cdot x^{m-r} \\
& \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
& + (p_{m-1} \cdot h_{n-m+1} + p_m \cdot h_{n-m}) \cdot x \\
& + p_m \cdot h_{n-m+1} .
\end{aligned}
$$

Thus the proposition is completely established.

3. The following extensions of the foregoing lemma, particularly in the case of a quadratic, may be worthy of notice.

If α, β be roots of $x^2 = px + q$, then

$$h_n = p \cdot h_{n-1} + q h_{n-2} \quad . \quad . \quad . \qquad (1),$$

where h_n is the sum of the homogeneous products of α, β of n dimensions.

Repeating the principle contained in (1) we find

$$
\begin{aligned}
h_n & = p^2 \cdot h_{n-2} + 2pq \cdot h_{n-3} + q^2 \cdot h_{n-4} \\
& = (p+q) \cdot [h]^{n-2}_{n-4} \text{ symbolically} \quad . \quad . \qquad (2).
\end{aligned}
$$

Applying the same principle to (2), we shall also get

$$h_n = p^3 \,.\, h_{n-3} + 3p^2q \,.\, h_{n-4} + 3pq^2 \,.\, h_{n-5} + q^3 \,.\, h_{n-6}$$
$$= (p+q)^3 \,.\, [h]^{n-3}_{n-6} \,;$$

and in like manner

$$= (p+q)^4 \,.\, [h]^{n-4}_{n-8}$$
$$= \qquad .$$

while generally

$$h_n = p_r \,.\, h_{n-r} + rp^{r-1}q \,.\, h_{n-r-1} + \frac{r(r-1)}{1\,.\,2} \,.\, p^{r-2}q^2 \,.\, h_{n-r-2}$$
$$+ \ldots + q^r \,.\, h_{n-2r}$$
$$= (p+q)^r \,.\, [h]^{n-r}_{n-2r} \text{ symbolically} \quad . \quad . \quad . \quad (3).$$

Again, h_n may be expressed under another form—in terms of the coefficients of the equation (p and q) only. Here it will be necessary to distinguish the cases n even and odd.

We have

$$h_1 = p \,,\; h_2 = ph_1 + q = p^2 + q \,,$$
$$h_3 = ph_2 + qh_1 = p^3 + 2pq \,.$$

By repeated applications of formula (1) we may deduce

$$h_8 = p^8 + 7p^6q + 15p^4q^2 + 10p^2q^3 + q^4 \,,$$
$$h_9 = p^9 + 8p^7q + 21p^5q^2 + 20p^3q^3 + 5pq^4 \,,$$
$$. \qquad . \qquad . \qquad . \qquad .$$

and generally

$$h_{2n} = p^{2n} + (2n-1) \,.\, p^{2n-2} \,.\, q + \frac{(2n-2)(2n-3)}{1\,.\,2} \,.\, p^{2n-4} \,.\, q^2$$
$$+ \frac{(2n-3)(2n-4)(2n-5)}{1\,.\,2\,.\,3} \,.\, p^{2n-6} \,.\, q^3 + \ldots + \frac{(n+1)n}{1\,.\,2} \,.\, p^2q^{n-1}$$
$$+ q^n \quad . \quad . \quad . \quad . \quad (4),$$

and

$$h_{2n+1} = p^{2n+1} + 2np^{2n-1}q + \frac{(2n-1)(2n-2)}{1\,.\,2} \,.\, p^{2n-3} \,.\, q^2$$
$$+ \frac{(2n-2)(2n-3)(2n-4)}{1\,.\,2\,.\,3} \,.\, p^{2n-5} \,.\, q^3 + \ldots + \frac{(n+2)(n+1)n}{1\,.\,2\,.\,3} \,.\, p^3q^{n-1}$$
$$+ (n+1)pq^n \quad . \quad . \quad (5),$$

each of which series consists of $n+1$ terms.

Conversely, the sums of the series (4) and (5) are known; for they are respectively h_{2n} and h_{2n+1}, that is

$$(\alpha^{2n+1} - \beta^{2n+1}) \div (\alpha - \beta)$$

and

$$(\alpha^{2n+2} - \beta^{2n+2}) \div (\alpha - \beta) .$$

4. To obtain results corresponding to (3) in the case of a cubic and of equations of a higher degree, we observe that in the case of a quadratic the number of terms in $(p+q)^r$ is the same as the number of different h's represented by $[h]^{n-r}_{n-2r}$.

Taking the equation

$$x^3 = px^2 + qx + r$$

we have

$$h_n = p \,.\, h_{n-1} + qh_{n-2} + r \,.\, h_{n-3} \quad . \quad . \qquad (6),$$

where h_n denotes the sum of the homogeneous products of roots of the cubic. By the repeated application of (6) we deduce

$$\begin{aligned} h_n &= p^2 \,.\, h_{n-2} + 2pq \,.\, h_{n-3} + \ldots + r^2 \,.\, h_{n-6} \\ &= (p+q+r)^2 \,.\, [h]^{n-2}_{n-6} \text{ symbolically,} \end{aligned}$$

where we observe that h_{n-4} occurs twice.

Again, it will follow that

$$h_n = (p+q+r)^3 \,.\, [h]^{n-3}_{n-9} ,$$

the number of terms in $(p+q+r)^3$ being ten, while there are only seven different h's, h_{n-5}, h_{n-6}, h_{n-7} each occurring twice.

So it will follow that

$$\begin{aligned} h_n &= (p+q+r)^4 \,.\, [h]^{n-4}_{n-12} \\ &= (p+q+r)^5 \,.\, [h]^{n-5}_{n-15} \\ &= \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}$$

and generally

$$h_n = (p+q+r)^r \,.\, [h]^{n-}_{u-3r} ,$$

the number of different h's being $2r+1$, and so disposed that in the first pair each h occurs once, in the second pair each h occurs twice, in the third three times, and so on up to the $\frac{r}{2}$th pair, while the middle h, *i.e.*, h_{n-2r}, occurs $\frac{r}{2}+1$ times. The same law holding when we reckon from the end, the total number of terms will be

$$4\left(1+2+3+\ldots+\frac{r}{2}\right)+\frac{r}{2}+1\,,$$

that is, $\frac{1}{2}(r+1)(r+2)$, which is the number of terms in $(p+q+r)^r$.

Again, if we consider the equation

$$x^4=px^3+qx^2+rx+s\,,$$

we have

$$h_n=p\,.\,h_{n-1}+q\,.\,h_{n-2}+r\,.\,h_{n-3}+s\,.\,h_{n-4}\,,$$

from which it may be deduced in the same manner as before that

$$\begin{aligned}h_n&=p^2\,.\,h_{n-2}+2pq\,.\,h_{n-3}+\ldots+s^2\,.\,h_{n-8}\\&=(p+q+r+s)^2\,.\,[h]^{n-2}_{n\;\;8}\text{ symbolically,}\end{aligned}$$

in which h_{n-4}, h_{n-5}, h_{n-} occur twice.

So it will follow that

$$\begin{aligned}h_n&=(p+q+r+s)^3\,.\,[h]^{n\;\;3}_{n-12}\\&=(p+q+r+s)^4\,.\,[h]^{n\;\;4}_{n\;\;16}\\&=\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\end{aligned}$$

and generally

$$h_n=(p+q+r+s)^r\,.\,[h]^{n-r}_{n-4r}\,,$$

the number of different h's being $3r+1$, which are repeated by the same law in going from the beginning and the end, and such that the sum of the number of all their coefficients is equal to the sum of the series,

$$1+3+6+10+\ldots+\frac{1}{2}(r+1)(r+2)$$

that is

$$\frac{(r+1)(r+2)(r+3)}{1\,.\,2\,.\,3}\,,$$

which is the number of terms in $(p+q+r+s)^r$.

Finally, in the case of the equation

$$x^m = p_1 x^{m-1} + p_2 . x^{m-2} + p_3 x^{m-3} + \dots + p_m ,$$

since

$$h_n = p_1 . h_{n-1} + p_2 . h_{n-2} + p_3 . h_{n-3} + \dots + p_m . h_{n-m} ,$$

it will follow in like manner that

$$\begin{aligned} h_n &= (p_1 + p_2 + p_3 + \dots + p_m)^2 . [h]^{n-2}_{n-m.2} \\ &= (\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad)^3 . [h]^{n-3}_{n-m.3} \\ &= (\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad) . [h]^{n-4}_{n-m.4} \\ &= \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}$$

and generally

$$h_n = (p_1 + p_2 + p_3 + \dots + p \ )^r . [h]^{n-r}_{n-mr} ,$$

the number of different h's being $(m-1)r+1$, which are repeated by the same law in going from the beginning and the end, and such that the sum of the number of all their coefficients is equal to the sum of the series,

$$1 + \frac{m-1}{1} + \frac{(m-1)m}{1.2} + \frac{(m-1)m(m+1)}{1.2.3} + \dots$$
$$+ \frac{(r+1)(r+2)(r+3)\dots(r+m-2)}{1.2.3\dots \ (m-2)} ,$$

that is

$$\frac{(r+1)(r+2)(r+3)\dots(r+m-1)}{1.2.3\dots \ (m-1)} ,$$

which is the number of terms in

$$(p_1 + p_2 + p_3 + \dots + p_m)^r .$$